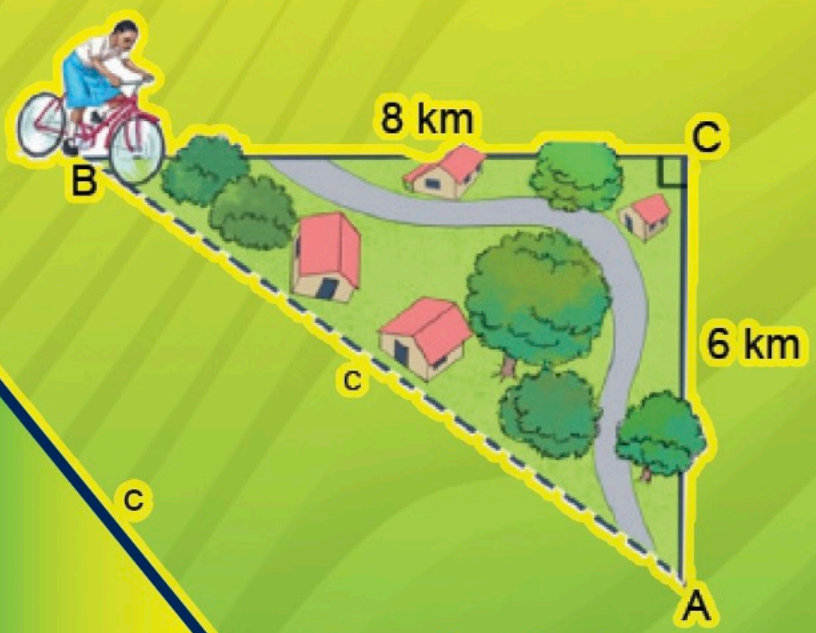
Mathematics

Pupil's Book

Standard Seven



$$c^2 = a^2 + b^2$$

Tanzania Institute of Education

Mathematics

Pupil's Book

Standard Seven

THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION,
SCIENCE AND TECHNOLOGY

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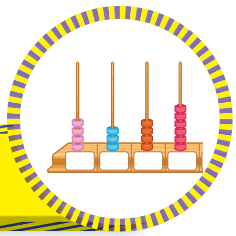
Dr Aneth A. Komba
Director General
Tanzania Institute of Education

Introduction

This is the fifth book in a series of five textbooks on Mathematics for Primary Schools from Standard Three to Seven. The book is prepared according to the 2020 Mathematics Syllabus for Primary Schools issued by the Ministry of Education, Science and Technology. This book consists of 13 chapters which are: Whole numbers, Addition and subtraction of whole numbers, Multiplication of whole numbers, Division of whole numbers, BODMAS, Ratios, Exponents and square roots of numbers, Fractions and decimals, Algebra, Speed, Pythagoras theorem, Business arithmetic, and Statistics. The book contains activities, illustrations and exercises which enhance learning. You are encouraged to do all activities and exercises in this book together with other assigments provided by your teacher.

Chapter One

Whole numbers



Introduction

In Standard Six, you learnt counting, identifying place values, total values, expanded form, reading, and writing whole numbers that do not exceed ten million (10 000 000). In this chapter, you will continue to learn counting, identifying place values, total values, expanded form, reading, and writing whole numbers that do not exceed one billion (1 000 000 000). The competencies gained will enable you to apply the concept of whole numbers in daily life activities such as measurements, money transactions, and business.

Exercise 1: Revision

Answer the following questions:

1. Write the following numbers in words:
   (a) 4 990 112
   (b) 9 952 067
   (c) 2 138 007
   (d) 9 215 310
   (e) 1 327 689

2. Write the following numbers in numerals:
   (a) Eight million one hundred sixty thousand one hundred and forty-five.
   (b) Five million four hundred eighty thousand eight hundred and twenty-two.
   (c) Seven million nine hundred thousand.
   (d) Six million seven hundred seventy thousand five hundred and eleven.
   (e) Three million two hundred thirty thousand two hundred and fifty-eight.

3. Write the place value of 7 in each of the following numbers:

(a) 1 748 426 (b) 7 629 031 (c) 8 514 367
(d) 9 819 766 (e) 3 897 895

4. Write in numerals the numbers formed by the following place values:

(a) Two millions, four hundred thousands, six ten thousands, five thousands, four hundreds, seven tens, and one ones.
(b) One millions, zero hundred thousands, eight ten thousands, eight thousands, zero hundreds, one tens, and five ones.
(c) Six millions, three hundred thousands, seven ten thousands, zero thousands, five hundreds, six tens, and eight ones.
(d) Nine millions, eight hundred thousands, five ten thousands, nine thousands, seven hundreds, zero tens, and one ones.
(e) Four millions, zero hundred thousands, seven ten thousands, two thousands, five hundreds, two tens, and four ones.

5. Write the following numbers in their expanded form:

(a) 4 836 712
(b) 2 374 986
(c) 9 500 821
(d) 8 812 093
(e) 7 009 285

6. If each of the digits 3, 7, 9, 6, 5, 8, and 1 is used once, write in numerals the number formed by the following statements:

(a) The largest number formed by the digits.
(b) The smallest number formed by the digits.

7. Use the numbers 4 183 763, 5 407 132, and 3 718 456 to answer the following questions:

(a) Which number has 6 in the tens place?
(b) Which number has 4 in the millions place?
(c) Which number has 1 in the hundreds place?
(d) Which number has 7 in the hundred thousands place?
(e) Which number has 3 in the ones place?

8. Carefully study the following sequences of numbers and fill in the blanks:

(a) 5 677 890, 5 677 892, 5 677 894, _____, _____, _____
(b) 6 600 600, 6 600 700, 6 600 800, _____, _____, _____
(c) _____, _____, 4 700 000, 4 600 000, 4 500 000, _____
(d) _____, _____, _____, 3 333 470, 3 333 465, 3 333 460
(e) _____, _____, 7 940 000, 7 950 000, _____, _____

9. Write a sequence of numbers in numerals from five million seven hundred thousand to six million by adding fifty thousand successively.

10. Write the following numbers in short form:

(a) 8 000 000 + 400 000 + 10 000 + 0 + 100 + 30 + 9
(b) 5 000 000 + 600 000 + 0 + 3 000 + 400 + 10 + 5
(c) 3 000 000 + 700 000 + 40 000 + 5 000 + 200 + 40 + 1
(d) 7 000 000 + 500 000 + 10 000 + 4 000 + 700 + 90 + 0
(e) 1 000 000 + 0 + 0 + 5 000 + 0 + 70 + 2

Counting in groups

Counting objects not exceeding one billion (1 000 000 000) in groups of ten million

Counting ten million shillings (10 000 000) formed by a bundle of one thousand banknotes of ten thousand shillings.

Figure 1: *A bundle of one thousand banknotes of ten thousand shillings with a value of ten million*

When you count ten bundles of ten million shillings each (or when you count 10 000 banknotes of ten thousand shillings), you will form a bundle with a total of one hundred million (100 000 000).

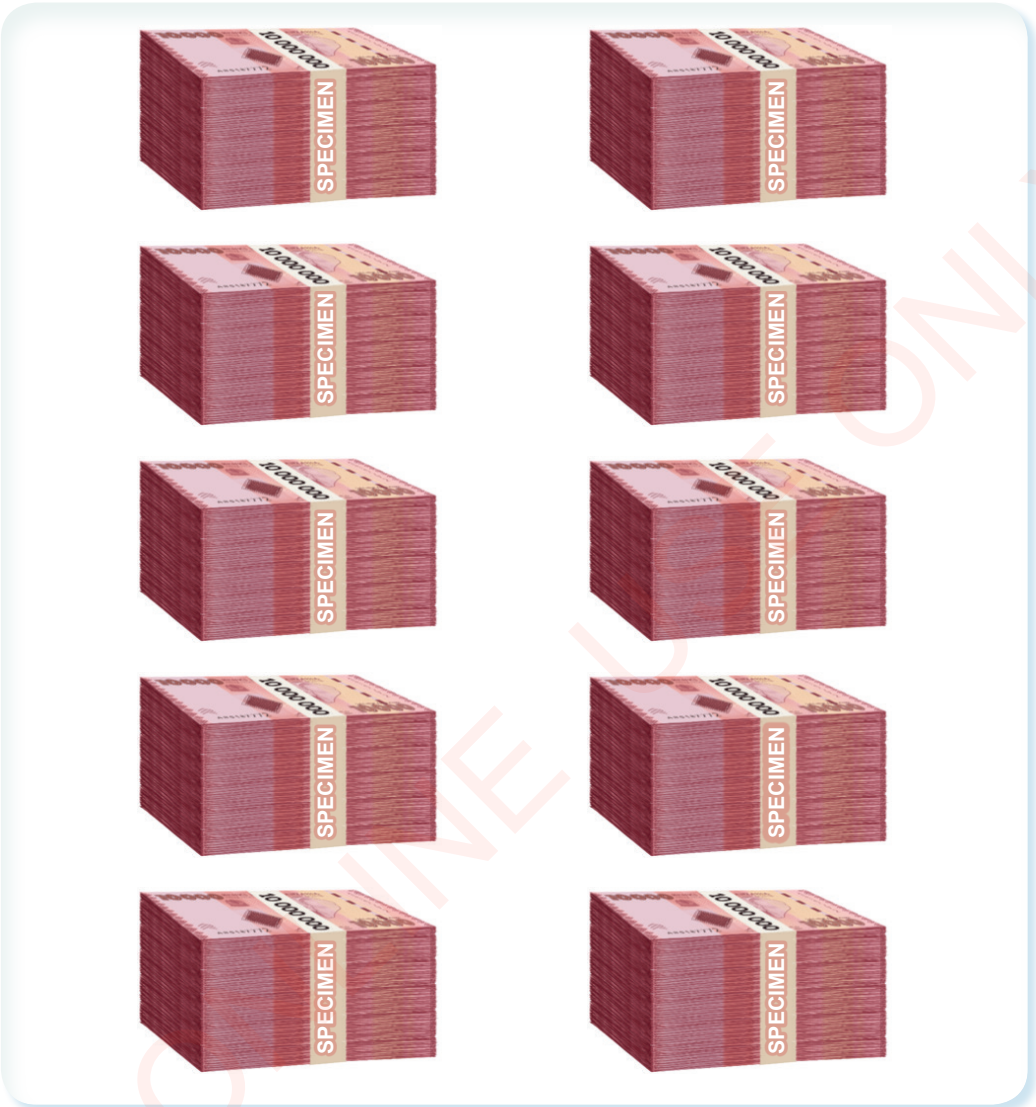


Figure 2: *Ten bundles of ten million shillings each with the value of one hundred millions*

In numerals, one hundred million is written as 100 000 000, and in words it is written as one hundred million. The number 10 000 000 is formed using eight digits, whereas 100 000 000 is formed using nine digits.

When you count ten bundles of one hundred million shillings each (or when you count 100 000 banknotes of ten thousand shillings), you will form bundles with a total of one billion shillings. In numerals, this number is written as 1 000 000 000 and in words, it is written as one billion. The number 100 000 000 is formed using nine digits, but 1 000 000 000 is formed using ten digits.

The following activities will help you to understand how to count objects in groups up to one billion easily:

Activity 1: Counting objects in groups of one hundred million using number cards

Carefully study the number cards in Figure 3, and then do the activities in each of the following steps:

Steps

1. Count four cards, and then write the total number of the values on the cards in numerals and in words.
2. Count six cards, and then write the total number of the values on the cards in numerals and in words.
3. Count eight cards, and then write the total number of the values on the cards in numerals and in words.
4. Count all the cards, and then write the total number of the values on the cards in numerals and in words.

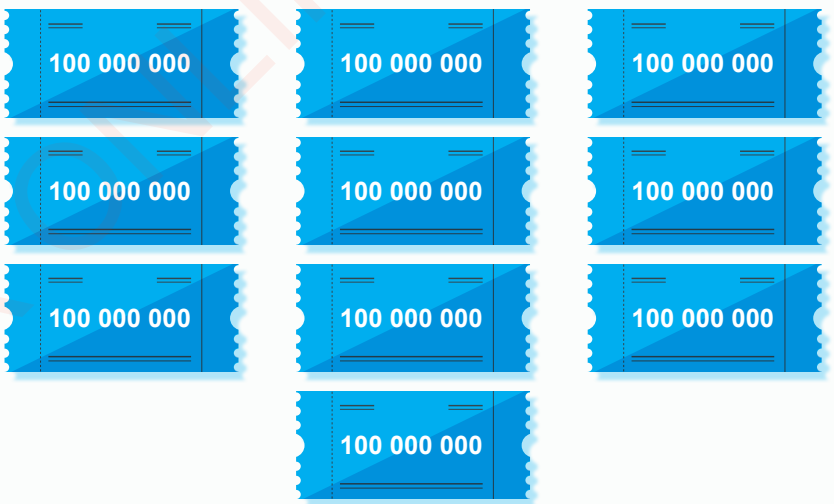


Figure 3: *Number cards each with a value of one hundred million*

Activity 2: Counting up to one billion using groups of one hundred million objects

A businessperson packed soap bars in containers. Each container was packed with one hundred million soap bars as shown in Figure 4.

Steps

1. Count five containers, and then write the sum of soap bars in the containers in numerals and in words.
2. Count six containers, and then write the sum of soap bars in the containers in numerals and in words.
3. Count nine containers, and then write the sum of soap bars in the containers in numerals and in words.
4. Count all the containers, and then write the sum of soap bars in the containers in numerals and in words.
5. Suppose the businessperson decides to pack 10 000 000 soap bars in each container. How many containers are required to pack a sum of soap bars equal to the answer in step 4?

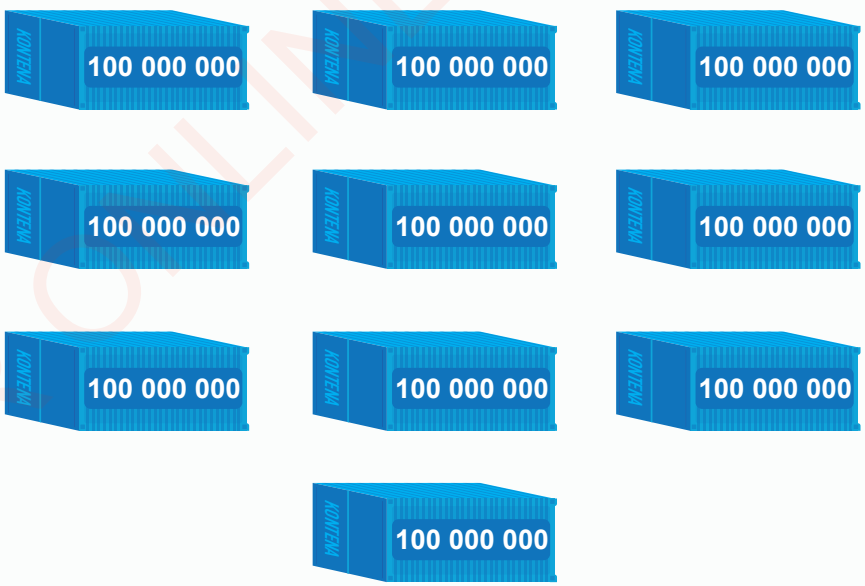


Figure 4: *Containers each packed with one hundred million soap bars*

Exercise 2

Answer the following questions:

1. How many ten millions will you count from three hundred million to four hundred million?
2. How many five millions will you count from one hundred million to four hundred million?
3. How many one hundred millions will you count from three hundred million to nine hundred million?
4. How many two hundred millions are there in eight hundred million?
5. How many three hundred millions will you count from one hundred million to one billion?
6. How many fifty millions will you count from three hundred million to eight hundred million?
7. A factory packed exercise books in containers for distributing in different places. Figure 5 shows the number of exercise books packed in each container. Count in groups the number of exercise books in the containers in items (a) to (e).

(a)

(b)

(c)

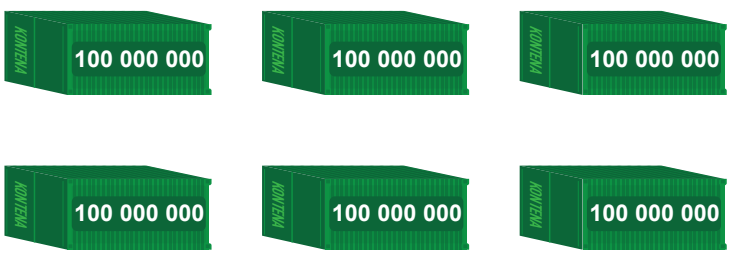
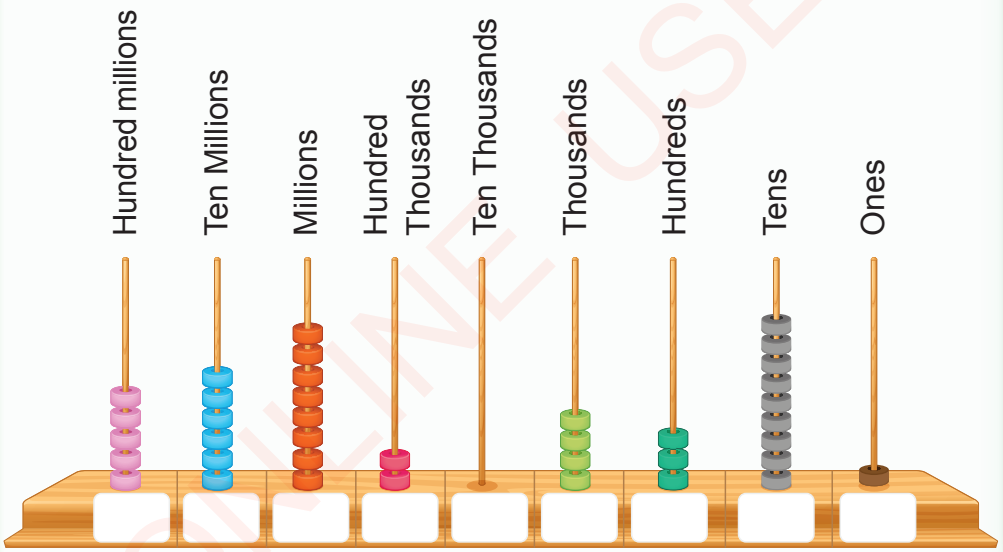


Figure 5: *Containers each packed with one hundred million exercise books*

8. Count and write in the boxes the digits of the number represented by the counters in the following abacus:



Place values and total values of digits in numbers not exceeding 1 000 000 000

One billion or 1 000 000 000 in numerals has ten digits. Also, any whole number between ten million and one billion is formed using eight or nine digits. In this section, you will learn how to identify place values of digits in numbers that do not exceed one billion. You will also learn how to identify the total values of digits of such numbers.

The place value and total value of a digit in a number depend on the position of the digit in the given number. The total value of a digit increases from the right to the left corresponding to the place value of the digit in the number. First, identify ones, followed by tens, hundreds, thousands, ten thousands, and so on depending on the number of digits in the given number.

Activity 3: Using number cards to show the total values of digits in a number

Instructions

Follow the instructions that will be provided by your teacher to do this activity with your fellow pupils.

Example 1

Arrange the digits of 1 000 000 000 in their corresponding place values.

Answer

The number 1 000 000 000 has ten digits. Thus, it has ten positions of place values. The place value of each digit in 1 000 000 000 is as follows:

Place value of a digit									
Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
1	0	0	0	0	0	0	0	0	0

You can also arrange the digits in their place values as follows:
1 is in billions
0 is in hundred millions
0 is in ten millions
0 is in millions
0 is in hundred thousands
0 is in ten thousands
0 is in thousands
0 is in hundreds
0 is in tens, and
0 is in ones.

Example 2

Write 912 834 567 by showing:

(a) The place value of each digit

(b) The total value of each digit

Answer

Number	**Digit**	(a) **Place value of a digit**	(b) **Total value of a digit**
912 834 567	9	Hundred millions	900 000 000
	1	Ten millions	10 000 000
	2	Millions	2 000 000
	8	Hundred thousands	800 000
	3	Ten thousands	30 000
	4	Thousands	4 000
	5	Hundreds	500
	6	Tens	60
	7	Ones	7

Example 3

Write the place value of 6 and the total value of 4 in 678 945 231.

Solution

The following table shows the place value of each of the digits in 678 945 231.

Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
6	7	8	9	4	5	2	3	1

Therefore, the place value of 6 in 678 945 231 is hundred millions.
The total value of 4 in 678 945 231 is 40 000.

Example 4

Write the following numbers in their expanded form:

(a) 789 273 453

(b) 132 261 514

Solution

Recall that, to write a number in its expanded form is to write it using the total values of each of its digits. Each total value is joined using addition operation.

(a) 789 273 453 = 700 000 000+ 80 000 000 + 9 000 000+200 000 + 70 000 + 3 000 + 400 + 50 + 3

(b) 132 261 514 = 100 000 000 + 30 000 000 + 2 000 000 + 200 000 + 60 000 + 1 000 + 500 + 10 + 4

Exercise 3

Answer the following questions:

1. Fill in the blanks in the following table:

Number	Place value of a digit									
	Billions	Hundred millions	Ten millions	Millions	Hundrend thousands	Ten thousands	Thousands	Hudndreds	Tens	Ones
230 821 834		2		0		2		8		4
45 903 475					9		3		7	5
154 368 210		1		4		6	8		1	
264 105 275				4		0		2		5
783 796 534			8		7		6		3	
22 536 907				2		3			0	7
63 731 425					7		1	4	2	
956 200 512		9		6		0		5	1	2
999 999 999					9			9	9	
1000 000 000				0		0		0		0

2. Write the following numbers in their expanded form:

(a) 978 203 322
(b) 231 378 680
(c) 457 105 275
(d) 858 546 124
(e) 267 536 907

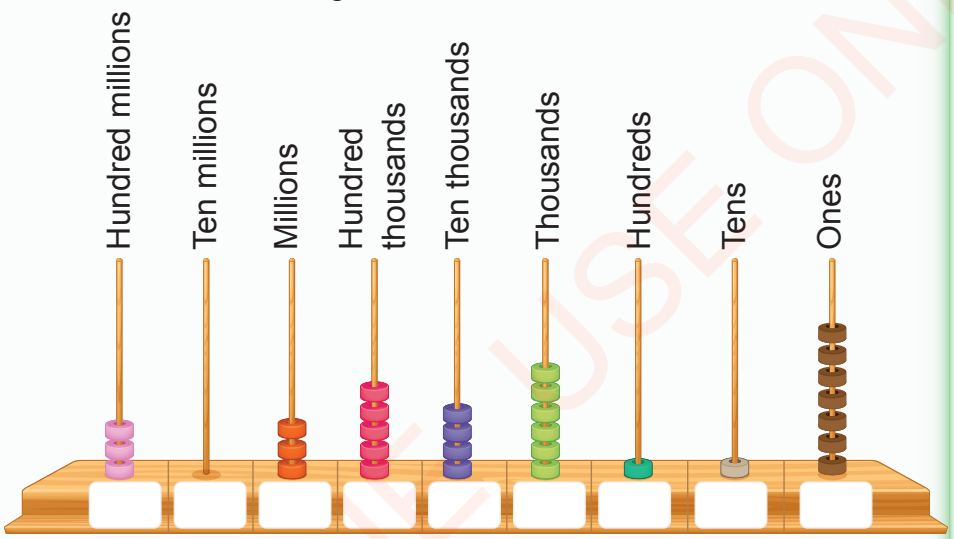
3. Write the place value of the underlined digit in each of the following numbers:

(a) 897 <u>4</u>34 291
(b) 8<u>7</u>3 226 675
(c) 94<u>3</u> 218 501
(d) <u>2</u>12 475 314
(e) 521 496 <u>6</u>54

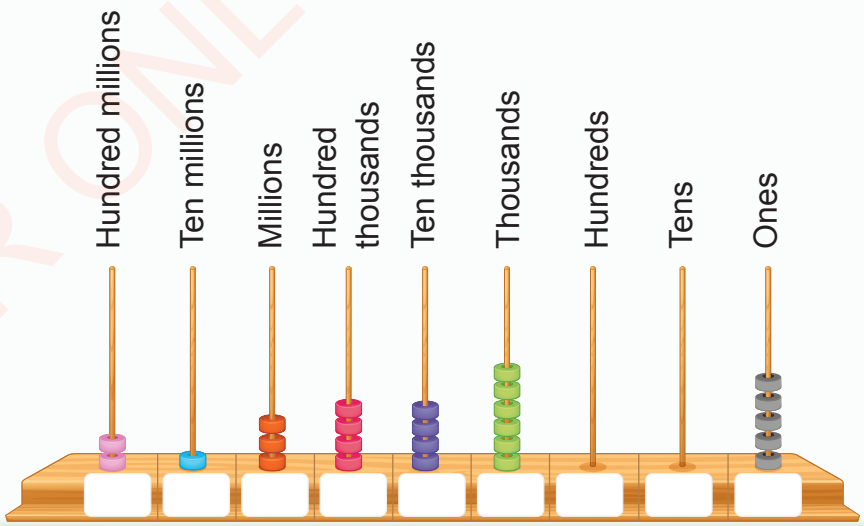
4. Write the total value of the underlined digit in each of the following numbers:

(a) 89<u>7</u> 434 291
(b) 8<u>7</u>3 226 675
(c) 943 2<u>1</u>8 501
(d) <u>2</u>12 475 314
(e) 521 <u>4</u>96 654

5. Write in the boxes the digits of the number represented by the counters in the following abacus:



6. Write in the boxes the digits of the number represented by the counters in the following abacus:



Reading numbers not exceeding one billion (1 000 000 000)

Reading a whole number depends on the number of digits and the total value of each digit in the given number. Start to identify ones, tens, hundreds, thousands, ten thousands, hundred thousands, and so on. Read the whole number from the left towards the right by considering the total value of the digit. In other words, start with a digit in the highest place value. For example, start with billions, followed by hundred millions, ten millions, millions, hundred thousands, ten thousands, thousands, hundreds, tens, and ones. Remember that, reading a number in numerals is the same as reading it in words.

Example 1

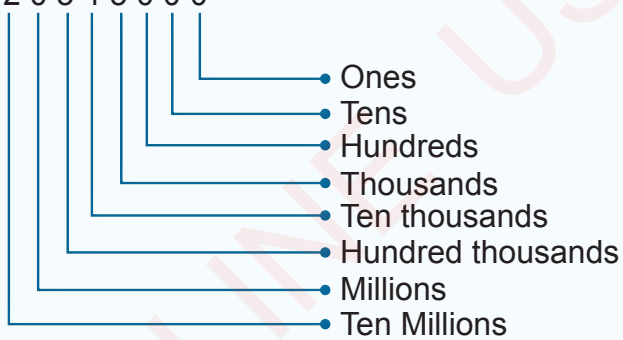
Read the following numbers:

(a) 20 345 000

(b) 654 987 432

Solution

(a) 2 0 3 4 5 0 0 0



Therefore, 20 345 000 is read as twenty million three hundred and forty-five thousand.

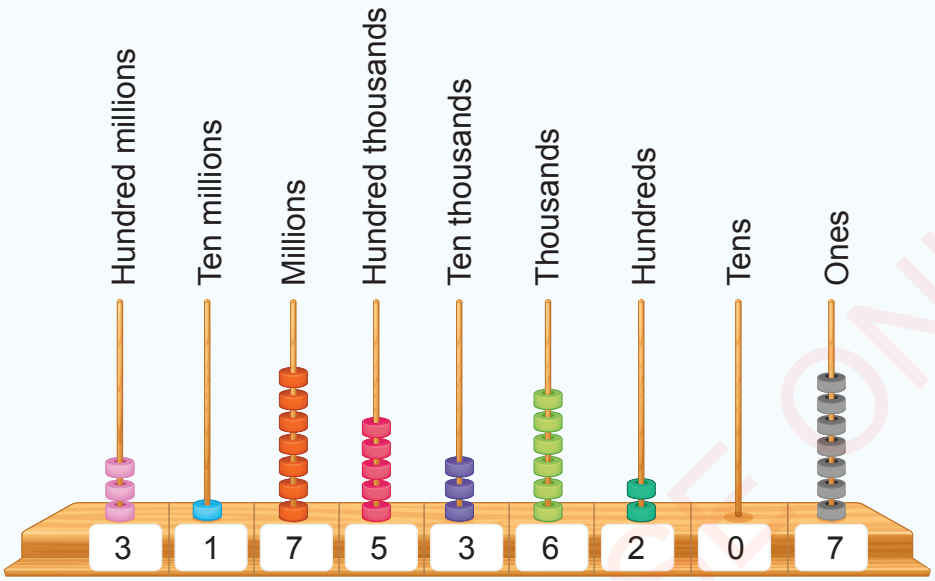
(b) 654 987 432

Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
6	5	4	9	8	7	4	3	2

Therefore, 654 987 432 is read as six hundred fifty-four million nine hundred eighty-seven thousand four hundred and thirty-two.

Example 2

Read the number in the boxes represented by the following abacus:



Answer

The number represented by the abacus is 317 536 207 and it is read as three hundred seventeen million five hundred thirty-six thousand two hundred and seven.

Example 3

Read the numbers in the following table:

Number in numerals	Number in words
11 000 340	Eleven million three hundred and forty
23 104 050	Twenty-three million one hundred four thousand and fifty
237 650 920	Two hundred thirty-seven million six hundred fifty thousand nine hundred and twenty
858 790 779	Eight hundred fifty-eight million seven hundred ninety thousand seven hundred and seventy-nine
704 999 456	Seven hundred four million nine hundred ninety-nine thousand four hundred and fifty-six

Exercise 4

Answer the following questions:

1. Read the following numbers:

(a) (i) 999 999 999 (ii) 15 000 000 (iii) 71 000 001
(iv) 156 000 002 (v) 11 000 003 (vi) 219 000 004

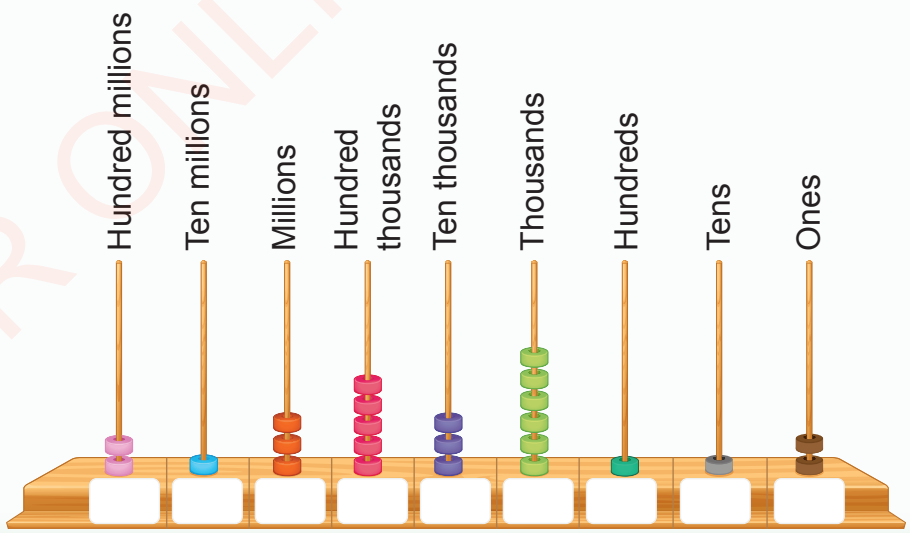
(b) (i) 41 000 005 (ii) 134 555 018 (iii) 16 000 015
(iv) 91 890 020 (v) 109 086 025 (vi) 601 000 030

(c) (i) 145 678 010 (ii) 17 056 020 (iii) 801 111 035
(iv) 144 022 123 (v) 15 999 543 (vi) 17 000 060

2. Read and write in words the numbers in the following table:

Number in numerals	Number in words
(a) 233 843 564	
(b) 62 402 400	
(c) 76 666 666	
(d) 978 457 813	
(e) 102 000 000	

3. Count the counters in the following abacus, and then answer the questions that follow:



(a) Write in the boxes the digits of the number represented by the counters in the abacus.
(b) Read the number represented by the abacus.
(c) Write in words the number represented by the abacus.

Writing whole numbers not exceeding one billion (1 000 000 000) in numerals and in words

A whole number can be written in numerals or in words. A number in numerals shows all the digits forming the number and their place values.

Example

Write the place value of each digit in the following numbers, and then write each number in words:
(a) 11 435 978
(b) 345 783 156
(c) 120 073 856

Answer

(a) The place value of each digit in 11 435 978 is as follows:

1 is in ten millions
1 is in millions
4 is in hundred thousands
3 is in ten thousands
5 is in thousands
9 is in hundreds
7 is in tens
8 is in ones

Therefore, 11 435 978 in words is eleven million four hundred thirty-five thousand nine hundred and seventy-eight.

(b) The place value of each digit in 345 783 156 is as follows:

3 is in hundred millions
4 is in ten millions
5 is in millions
7 is in hundred thousands
8 is in ten thousands
3 is in thousands
1 is in hundreds
5 is in tens
6 is in ones

Therefore, 345 783 156 in words is three hundred forty-five million seven hundred eighty-three thousand one hundred and fifty-six.

(c) The place value of each digit in 120 073 856 is as follows:

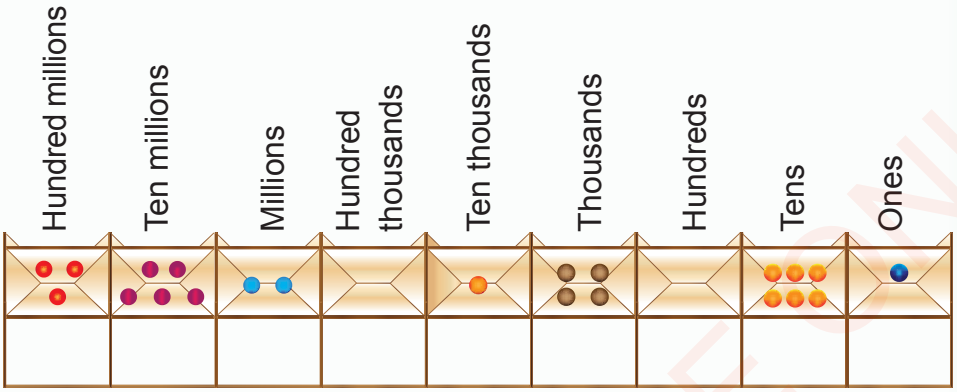
1 is in hundred millions
2 is in ten millions
0 is in millions
0 is in hundred thousands
7 is in ten thousands
3 is in thousands
8 is in hundreds
5 is in tens
6 is in ones

Therefore, 120 073 856 in words is one hundred twenty million seventy-three thousand eight hundred and fifty-six.

Exercise 5

Answer the following questions:

1. Count the counters in the following number basin, and then answer the questions that follow:



   (a) Write the number of counters in the blank spaces provided.
   (b) Write the number obtained in (a) in numerals.
   (c) Write the number obtained in (b) in words.

2. Write the following numbers in words:
   (a) 19 342 057
   (b) 123 256 080
   (c) 333 437 000
   (d) 562 870 074
   (e) 1 000 000 000

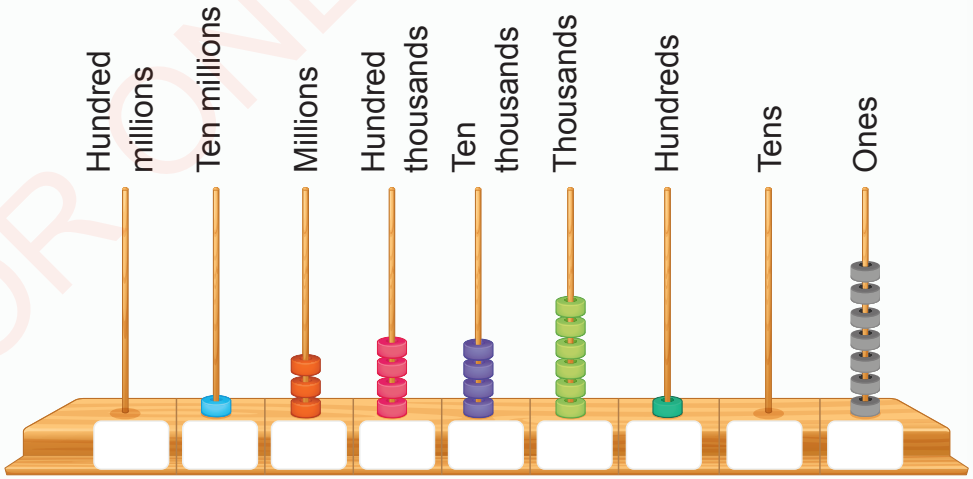
3. Write the following numbers in numerals:
   (a) Eight hundred twenty million nine hundred ninety thousand five hundred and seventy
   (b) Forty-five million eight hundred seventy-nine thousand and five hundred
   (c) Nine hundred twenty-one million one hundred twenty-four thousand two hundred and forty-four
   (d) Seventy-seven million six hundred seventy-four thousand eight hundred and eighty-five
   (e) Seventy-eight million six hundred and forty-six

4. Write the place value of 6 in each of the following numbers:
(a) 11 645 218
(b) 55 446 128
(c) 534 460 281
(d) 236 381 483
(e) 88 321 610

5. Write the total value of 6 in each of the following numbers:
(a) 11 645 218
(b) 55 446 128
(c) 534 460 281
(d) 236 381 483
(e) 88 321 610

6. Write the following numbers in their expanded form. Item (a) is done as an example:
(a) 55 301 106 = 50 000 000 + 5 000 000 + 300 000 + 0 + 1 000 + 100 + 0 + 6
(b) 257 486 120
(c) 561 074 508
(d) 48 051 900
(e) 29 109 239

7. Study carefully the following abacus, and then answer the questions that follow:



(a) Write in the boxes the digits of the number represented by the abacus.
(b) Write in numerals the number represented by the abacus.
(c) Write in words the number represented by the abacus.

Revision exercise

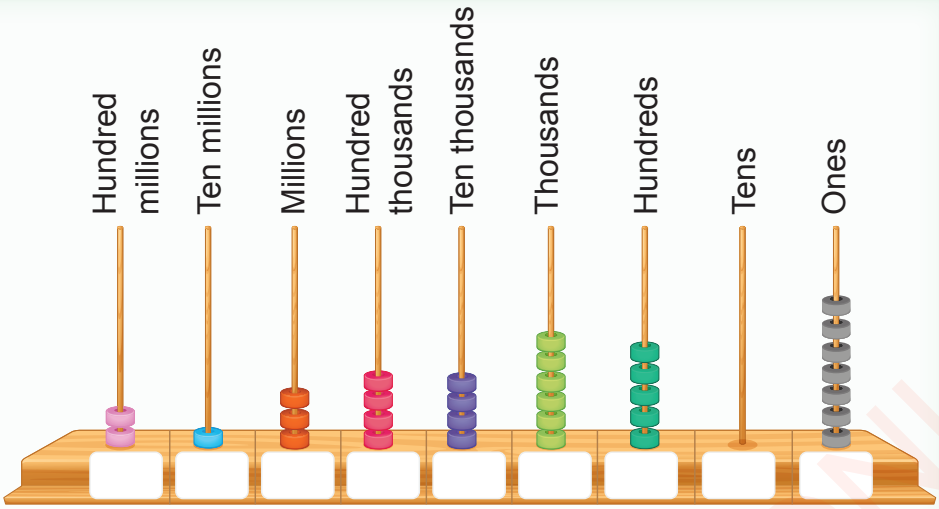
Answer the following questions:

1. Write the next three numbers in the following sequences by successively adding one million:
   (a) 13 673 800, _____, _____, _____
   (b) 45 318 166, _____, _____, _____
   (c) 237 105 200, _____, _____, _____
   (d) 568 245 000, _____, _____, _____
   (e) 31 003 890, _____, _____, _____

2. Write the next three numbers in the following sequences by successively subtracting five hundred thousand:
   (a) 53 500 000, _____, _____, _____
   (b) 554 219 585, _____, _____, _____
   (c) 63 862 000, _____, _____, _____
   (d) 17 165 140, _____, _____, _____
   (e) 235 703 000, _____, _____, _____

3. Write the next numbers by adding successively thirty thousand from 30 800 000 to 31 010 000.

4. Study carefully the following abacus, and then write the number represented in:
   (a) Numerals
   (b) Words



5. Fill in the blanks in the following sequences by successively subtracting one hundred:

(a) 19 153 235, _____, _____, _____
(b) 73 000 000, _____, _____, _____
(c) 33 995 768, _____, _____, _____
(d) 125 813 831, _____, _____, _____
(e) 222 183 216, _____, _____, _____

6. In a certain country, the population is 15 000 000. If the population increases by 1 000 000 every year, what will be the population of the country after five years?

7. Study carefully the following table, and then fill in the blanks to show the place values of digits in the given numbers:

Number	Place value of a digit								
	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
645 834 821									
989 967 093									
244 101 368									
979 807									
99 687 345									

8. Write the following numbers in their expanded form:
(a) 938 475 930
(b) 436 138 102
(c) 71 054 257
(d) 100 968 543
(e) 675 362 079

9. Write the total value of the underlined digit in the following numbers:
(a) 3̲63 142 914
(b) 26 738 5̲10
(c) 12 2̲62 189
(d) 76 541̲ 341
(e) 524̲ 755 964

10. Write the place value of the underlined digit in the following numbers:
(a) 16̲ 923 041
(b) 2̲37 951 830
(c) 83 26̲9 218
(d) 2̲6 501 314
(e) 7̲21 554 946

11. Write the following numbers in words:
(a) 34 729 075
(b) 420 548 560
(c) 644 488 379
(d) 75 780 045
(e) 25 111 111

12. Write the following numbers in numerals:
(a) One hundred forty-five million two hundred and fourteen thousand
(b) Three hundred million eight hundred thousand
(c) Five hundred million and one
(d) Six hundred nineteen million six hundred thousand and sixty-six
(e) Twenty-two million ten thousand and two

13. Write the following numbers in short form:

(a) 600 000 000 + 30 000 000 + 0 + 500 000 + 40 000 + 3 000 + 100 + 20 + 9

(b) 80 000 000 + 7 000 000 + 600 000 + 0 + 5 000 + 400 + 30 + 2

(c) 700 000 000 + 90 000 000 + 2 000 000 + 100 000 + 80 000 + 0 + 0 + 0 + 0

14. Draw the abacuses to represent the place values of digits in the following numbers:

(a) 345 768 219

(b) 567 812 493

15. Draw a number basin to represent the place value of digits in the following numbers:

(a) 1 000 000 000

(b) 653 478 921

Summary

1. In order to count many objects between ten million and one billion easily, count the objects in groups of one million, ten million or one hundred million.
2. In order to read a whole number easily, first identify the place value of each digit in the number. Read the given number from left (the digit with the highest total value) towards the right.
3. When writing a number in its expanded form, start to write from the highest total value to the smallest.

Chapter Two

Addition and subtraction of whole numbers

Introduction

In Standard Six, you learnt addition and subtraction of whole numbers up to ten million (10 000 000). In this chapter, you will continue to learn about addition and subtraction of whole numbers whose sum does not exceed one billion (1 000 000 000). You will also solve word problems involving addition and subtraction. The competencies gained will help you to apply numbers in your day to day life. For example, in business, that is, selling and buying of commodities or doing different transactions.

Exercise 1. Revision

Answer the following questions:

1. 1 202 653 + 8 121 152 =

2. 3 222 789 + 2 364 387 =

3. 5 192 000 + 1 379 263 =

4. 6 236 108 + 618 293 =

5. 4 133 256 + 329 684 =

6.
$$\begin{array}{r} 6\,354\,360 \\ +\ 1\,345\,234 \\ \hline \\ \hline \end{array}$$

7.
$$\begin{array}{r} 4\,463\,548 \\ +\ 5\,234\,451 \\ \hline \\ \hline \end{array}$$

8.
$$\begin{array}{r} 7\,463\,854 \\ +\ 2\,443\,132 \\ \hline \\ \hline \end{array}$$

9.
$$\begin{array}{r} 663\,548 \\ +\ 8\,500\,101 \\ \hline \\ \hline \end{array}$$

10. 5 982 250 − 4 181 152 =

11. 7 121 683 − 200 757 =

12. 8 564 112 − 4 124 000 =

13. 6 526 231 − 958 744 =

14. 5 036 521 − 2 563 256 =

15. 9 463 548 − 6 234 451

16. 4 474 608 − 1 295 741

17. 5 505 835 − 1 334 651

18. 6 425 212 − 1 445 234

19. 5 463 548 + 1 251 642 + 2 234 451

20. 4 743 485 + 5 243 154

21. A factory manufactured 5 563 630 metres of fabric. If 2 137 789 metres of that fabric was sold, how many metres of fabric were left?

22. On the farmers' day, it was reported that, out of 7 126 789 farmers, 1 878 375 farmers cultivate food crops whereas the rest cultivate cash crops. How many farmers cultivate cash crops?

23. A company sold cars for three consecutive years as follows: 765 987 cars in 2006, 1 873 467 cars in 2007, and 765 345 cars in 2008. Find the total number of cars sold by the company in the three years.

24. A textile factory manufactured clothes as follows: 456 897 pairs of trousers, 864 234 shirts, and 786 345 jackets. Find the total number of clothes manufactured by the factory.

25. The number of livestock in a certain region in 2010 was: 2 092 450 cows, 612 576 goats, and 829 730 sheep. How many livestock were in that region in that year?

Addition of Whole Numbers

Activity 1: Addition of whole numbers using a counting basin

Use a counting basin to add 638 214 567 and 241 763 232 by using the following steps:

Steps

1. Prepare not less than 60 counters.
2. Put the counters in the counting basin by considering the place value of each digit for the first number (638 214 567).
3. Add the counters by considering the place value of each digit in the second number (241 763 232).
4. Count all the counters in the counting basin by considering the place value of each digit.
5. Write the answer starting with the number of counters on the side with the highest place value in the counting basin.

Addition of whole numbers arranged horizontally without regrouping

Example

321 164 423 + 127 322 410 =

Steps

1. Add ones: 3 + 0 = 3. Write 3 in the ones place.	321 164 423 + 127 322 410 = _____3
2. Add tens: 2 + 1 = 3. Write 3 in the tens place.	321 164 423 + 127 322 410 = _____33
3. Add hundreds: 4 + 4 = 8. Write 8 in the hundreds place.	321 164 423 + 127 322 410 = _____833

4. Add thousands: 4 + 2 = 6. Write 6 in the thousands place.	321 164 423 + 127 322 410 = _ _ _ _ _ 6 833
5. Add ten thousands: 6 + 2 = 8. Write 8 in the ten thousands place.	321 164 423 + 127 322 410 = _ _ _ _ 86 833
6. Add hundred thousands: 1 + 3 = 4. Write 4 in the hundred thousands place.	321 164 423 + 127 322 410 = _ _ _ 486 833
7. Add millions: 1 + 7 = 8. Write 8 in the millions place.	321 164 423 + 127 322 410 = _ _ 8 486 833
8. Add ten millions: 2 + 2 = 4. Write 4 in the ten millions place.	321 164 423 + 127 322 410 = _ 48 486 833
9. Add hundred millions: 3 + 1 = 4. Write 4 in the hundred millions place.	321 164 423 + 127 322 410 = 448 486 833
Therefore, 321 164 423 + 127 322 410 = 448 486 833.	

Addition of whole numbers arranged vertically without regrouping

Example

$$\begin{array}{r} 2\,3\,4\;\;4\,6\,3\;\;5\,4\,8 \\ +\;3\,4\,5\;\;2\,3\,4\;\;4\,5\,1 \\ \hline \\ \hline \end{array}$$

Solution

Using the vertical method, arrange the numbers by following the place values of each digit in each number as follows:

	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
	2	3	4	4	6	3	5	4	8
+	3	4	5	2	3	4	4	5	1
	5	7	9	6	9	7	9	9	9

Therefore,

$$\begin{array}{r} 2\,3\,4\,4\,6\,3\,5\,4\,8 \\ +\ 3\,4\,5\,2\,3\,4\,4\,5\,1 \\ \hline 5\,7\,9\,6\,9\,7\,9\,9\,9 \end{array}$$

Exercise 2

Answer the following questions:

1. 462 123 222 + 131 312 222 =

2. 671 146 133 + 123 653 433 =

3. 12 461 331 + 334 111 312 =

4. 246 213 344 + 421 321 443 =

5. 666 666 666 + 222 222 222 =

6. $\begin{array}{r} 5\,4\,1\,4\,1\,7\,4\,1\,1 \\ +\ 2\,3\,3\,2\,2\,2\,1\,1\,1 \\ \hline \\ \hline \end{array}$

7. $\begin{array}{r} 1\,3\,6\,4\,1\,2\,9\,7\,8 \\ +\ 2\,2\,0\,4\,4\,0\,0\,0\,0 \\ \hline \\ \hline \end{array}$

8. $\begin{array}{r} 5\,9\,3\,3\,7\,4\,4\,6\,4 \\ +\ 1\,0\,6\,3\,0\,2\,4\,0\,4 \\ \hline \\ \hline \end{array}$

9. $\begin{array}{r} 4\,6\,5\,4\,5\,6\,3\,1\,2 \\ +\ 5\,0\,2\,3\,0\,1\,4\,1\,4 \\ \hline \\ \hline \end{array}$

10. 546 117 411
+ 233 222 111

11. 13 457 316 + 423 321 123 =

12. 292 888 999 + 101 111 000 =

13. 776 667 845 + 223 332 154 =

14. 854 564 312 + 43 432 675 =

15. 714 411 514 + 173 548 251 =

16. 417 514 514
+ 230 420 171

17. 231 091 583
+ 206 707 314

18. 776 555 222
+ 213 333 444

19. 798 456 213
+ 1 223 473

20. 246 735 233
+ 433 243 132

Addition of whole numbers arranged horizontally by regrouping

Example

528 589 348 + 467 896 876 =

Steps

1. Add ones: 8 + 6 = 14. Write 4 in the ones place. Regroup 10 ones into 1 tens and add it to the tens place.	528 589 348 + 467 896 876 = _ _ _ _ _ _ _ _ 4

2. Add tens: 4 + 7 + 1 = 12. Write 2 in the tens place. Regroup 10 tens into 1 hundreds and add it to the hundreds place.	528 589 348 + 467 896 876 = _ _ _ _ _ _ _ 2 4
3. Add hundreds: 3 + 8 + 1 = 12. Write 2 in the hundreds place. Regroup 10 hundreds into 1 thousands and add it to the thousands place.	528 589 348 + 467 896 876 = _ _ _ _ _ _ 224
4. Add thousands: 9 + 6 + 1 = 16. Write 6 in the thousands place. Regroup 10 thousands into 1 ten thousands and add it to the ten thousands place.	528 589 348 + 467 896 876 = _ _ _ _ _ 6 224
5. Add ten thousands: 8 + 9 + 1 = 18. Write 8 in the ten thousands place. Regroup 10 ten thousands into 1 hundred thousands and add it to the hundred thousands place.	528 589 348 + 467 896 876 = _ _ _ _ 86 224

6. Add hundred thousands: 5 + 8 + 1 =14. Write 4 in the hundred thousands place. Regroup 10 hundred thousands into 1 millions and add it to the millions place.	528 589 348 + 467 896 876 =_ _ _ 486 224
7. Add millions: 8 + 7 + 1 = 16. Write 6 in the millions place. Regroup 10 millions into 1 ten millions and add it to the ten millions place.	528 589 348 + 467 896 876 = _ _ 6 486 224
8. Add ten millions: 2 + 6 + 1 = 9. Write 9 in the ten millions place.	528 589 348 + 467 896 876 =_ 9 6 486 224
9. Add hundred millions: 5 + 4 = 9. Write 9 in the hundred millions place.	528 589 348 + 467 896 876 = 996 486 224
Therefore, 528 589 348 + 467 896 876 = 996 486 224.	

Addition of whole numbers arranged vertically by regrouping

Example

$$\begin{array}{r} 4\,3\,1\;5\,4\,9\;7\,2\,8 \\ +\;3\,5\,6\;7\,2\,7\;1\,9\,6 \\ \hline \\ \hline \end{array}$$

Steps

Align the given numbers vertically by considering the place value of each digit as follows:

1. Add ones: 8 + 6 = 14. Write 4 in the ones place. Regroup 10 ones into 1 tens and add it to the tens place.	4 3 1 5 4 9 7 2 8 + 3 5 6 7 2 7 1 9 6 4
2. Add tens: 2 + 9 + 1 = 12. Write 2 in the tens place. Regroup 10 tens into 1 hundreds and add it to the hundreds place.	4 3 1 5 4 9 7 2 8 + 3 5 6 7 2 7 1 9 6 2 4
3. Add hundreds: 1 + 7 + 1 = 9. Write 9 in the hundreds place.	4 3 1 5 4 9 7 2 8 + 3 5 6 7 2 7 1 9 6 9 2 4
4. Add thousands: 9 + 7 = 16. Write 6 in the thousands place. Regroup 10 thousands into 1 ten thousands and add it to the ten thousands place.	4 3 1 5 4 9 7 2 8 + 3 5 6 7 2 7 1 9 6 6 9 2 4
5. Add ten thousands: 4 + 2 + 1 = 7. Write 7 in the ten thousands place.	4 3 1 5 4 9 7 2 8 + 3 5 6 7 2 7 1 9 6 7 6 9 2 4
6. Add hundred thousands: 5 + 7 = 12. Write 2 in the hundred thousands place. Regroup 10 hundred thousands into 1 millions and add it to the millions place.	4 3 1 5 4 9 7 2 8 + 3 5 6 7 2 7 1 9 6 2 7 6 9 2 4
7. Add millions: 1 + 6 + 1 = 8. Write 8 in the millions place.	4 3 1 5 4 9 7 2 8 + 3 5 6 7 2 7 1 9 6 8 2 7 6 9 2 4

8. Add ten millions: 3 + 5 = 8. Write 8 in the ten millions place.	4 3 1 5 4 9 7 2 8 + 3 5 6 7 2 7 1 9 6 8 8 2 7 6 9 2 4
9. Add hundred millions: 4 + 3 = 7. Write 7 in the hundred millions place.	4 3 1 5 4 9 7 2 8 + 3 5 6 7 2 7 1 9 6 7 8 8 2 7 6 9 2 4

Therefore,
4 3 1 5 4 9 7 2 8
+ 3 5 6 7 2 7 1 9 6
7 8 8 2 7 6 9 2 4

Exercise 3

Answer the following questions:

1. 221 113 446 + 539 473 644 =

2. 613 123 546 + 123 987 123 =

3. 714 154 141 + 213 896 641 =

4. 209 610 804 + 622 859 746 =

5. 9 637 + 13 222 709 + 2 364 387 =

6. 8 8 8 8 8 8 8 8 8
+ 1 1 1 1 1 1 1 1 2

7. 6 7 8 8 9 0 0 4 8
+ 2 4 6 7 6 7 3 9 6

8. 6 7 4 4 1 2 1 2 6
+ 2 2 1 6 7 8 1 9 2

9. 8 7 1 6 5 2 4 2 5
+ 1 2 7 7 5 2 5 4 2

10. 2 3 6 2 2 2 3 3 3
+ 6 4 6 4 4 3 9 9 9

11. 12 921 396 + 998 + 4 001 968 =

12. 677 777 666 + 888 229 =

13. 49 835 + 4 984 375 + 9 998 729 =

14. 231 546 645 + 854 667 =

15. 998 + 40 109 981 =

16.
$$\begin{array}{r} 342\,641\,982 \\ +\,524\,164\,298 \\ \hline \\ \hline \end{array}$$

17.
$$\begin{array}{r} 513\,912\,853 \\ +\,315\,219\,358 \\ \hline \\ \hline \end{array}$$

18.
$$\begin{array}{r} 238\,119\,365 \\ +\,643\,872\,456 \\ \hline \\ \hline \end{array}$$

19.
$$\begin{array}{r} 444\,555\,728 \\ +\,289\,453\,766 \\ \hline \\ \hline \end{array}$$

20.
$$\begin{array}{r} 478\,127\,712 \\ +\,513\,493\,109 \\ \hline \\ \hline \end{array}$$

Word problems on addition

Example 1

In a period of three consecutive years, Jackson earned 135 003 600 shillings in the first year, 40 634 000 shillings in the second year, and 123 350 350 shillings in the third year. How much money did he earn in the three years?

Solution

Income in the first year = 135 003 600 shillings
Income in the second year = 40 634 000 shillings
Income in the third year = 123 350 350 shillings
Add them in order to get the total income:

$$\begin{array}{rr} & \text{sh } 135\ 003\ 600 \\ & \text{sh } \ 40\ 634\ 000 \\ + & \text{sh } 123\ 350\ 350 \\ \hline & \text{sh } 298\ 987\ 950 \\ \hline \end{array}$$

Therefore, Jackson's total income for three years was 298 987 950 shillings.

Example 2

A certain country used 564 145 263 kilograms of sugar in April and 295 243 453 kilograms of sugar in May. How many kilograms of sugar did the country use in the two months?

Solution

Mass of sugar used in April = 564 145 263 kg

Mass of sugar used in May = 295 243 453 kg

To get the number of kilograms of sugar used in the two months, add the mass of sugar used in those months:

$$\begin{array}{rr} & \text{kg } 564\ 145\ 263 \\ + & \text{kg } 295\ 243\ 453 \\ \hline & \text{kg } 859\ 388\ 716 \end{array}$$

Therefore, the country used 859 388 716 kilograms of sugar in the two months.

Exercise 4

Answer the following questions:

1. The mass of fish caught by certain fishermen was 21 832 431 kilograms in the first year, 11 235 032 kilograms in the second year, and 13 145 212 kilograms in the third year. How many kilograms of fish were caught in the three years?

2. Farmers sold their cotton in five years with the following masses: 814 565 kg, 19 136 198 kg, 138 118 kg, 12 468 881 kg, and 134 659 895 kg. What was the total mass of cotton sold by the farmers?

3. The villagers made 13 893 123 burnt bricks in the first year and 10 432 023 burnt bricks in the second year. How many bricks were made in the two years?

4. Four buses were sold as follows: The first bus was sold for 123 800 000 shillings, the second bus was sold for 109 880 000 shillings, the third bus was sold for 99 742 000 shillings, and the fourth bus was sold for 138 750 000 shillings. Find the total amount of money earned.

5. A village spent some money for rehabilitation as follows: 23 403 100 shillings for two primary schools, 11 171 300 shillings for roads, 11 302 600 shillings for the dispensary, and 34 011 200 shillings for water pipes. Find the total amount of money spent for the rehabilitation.

6. Five schools in a certain ward launched a bricks production project for one year. The first school made 2 000 115 bricks, the second school made 3 000 031 bricks, the third school made 1 863 512 bricks, the fourth school made 2 024 100 bricks, and the fifth school made 1 110 040 bricks. How many bricks did the schools make altogether in that year?

7. A company paid monthly bills as follows: Electricity bill 5 781 800 shillings, water consumption bill 4 189 550 shillings, and car fuel bill 3 340 250 shillings. How much money did the company spend for all the bills?

8. Sports fans watched five games played in different grounds at the same time as follows: 23 400 320 football fans,1 302 420 athletic fans, 120 330 netball fans, 990 100 handball fans, and 2 234 876 basketball fans. Find the total number of fans who watched all the games.

9. Crops exported were 80 765 890 kilograms in July and 25 540 920 kilograms in August. How many kilograms of crops were exported in the two months?

10. In a certain country, the population in three regions was as follows: The first region had 6 924 233 people, the second region had 5 666 982 people, and the third region had 9 984 620 people. Find the total number of people in the three regions.

11. A soap manufacturing factory manufactured 15 234 190 bars in the first year, 26 450 200 bars in the second year, and 18 400 672 bars in the third year. How many bars were manufactured in the three years?

12. A company sold two plots of land for 42 850 000 shillings and 37 650 000 shillings, respectively. How much money did the company earn from the sales of the two plots?

13. There are 850 327 books in the first library and 1 204 645 books in the second library. Find the total number of books in the two libraries.

14. A company bought 4 764 092 cars in cash and 335 168 cars on credit. How many cars did the company buy altogether?

15. Zainab received a salary of 2 235 250 shillings. She was also given a loan of 958 525 shillings by her employer. How much money did she get in total?

16. Masoud sold his items as follows: a laptop for 1 160 500 shillings, a radio for 280 000 shillings, and a cupboard for 190 500 shillings. How much money did he get from the sales?

17. Agatha bought a sofa set for 785 985 shillings, a bed for 364 225 shillings, and a mattress for 246 352 shillings. Find the total amount of money she spent on all the items.

18. The income of a businessperson for last year was 42 628 500 shillings. If the businessperson borrowed 10 846 200 shillings from the bank, how much money did the businessperson get from both sources?

19. Two houses were sold for 53 639 450 shillings and 67 954 775 shillings respectively. Find the total price for the two houses.

20. A factory manufactured exercise books as follows: 161 985 478 exercise books in 2017, 148 216 592 exercise books in 2018, and 122 684 312 exercise books in 2019. Find the total number of exercise books manufactured.

Subtraction of whole numbers

Activity 2: Subtraction of whole numbers using a counting basin

Use the counting basin to subtract the numbers given by your teacher.

Steps

1. Prepare not less than 60 counters.
2. Put the counters in the counting basin by considering the place value of each digit of the number from which another number is subtracted.
3. Reduce the counters by considering the place value of each digit in the number to be subtracted.
4. Write the number of counters remaining in the counting basin by considering the place value of each digit.
5. Write the answer starting with the remaining counters in the counting basin on the side with the highest place value.

Subtraction of whole numbers arranged horizontally without regrouping

Example

8 67 8 77 842 − 4 36 846 701 =

Steps

1. Subtract ones: 2 – 1 =1. Write 1 in the ones place.	8 67 8 77 842 − 4 36 846 701 =_ _ _ _ _ _ _ _1

2. Subtract tens: 4 – 0 = 4. Write 4 in the tens place.	867 877 842 − 436 846 701 = _ _ _ _ _ _ _ 41
3. Subtract hundreds: 8 – 7 = 1. Write 1 in the hundreds place.	867 877 842 − 436 846 701 = _ _ _ _ _ _ 141
4. Subtract thousands: 7 – 6 = 1. Write 1 in the thousands place.	867 877 842 − 436 846 701 = _ _ _ _ _ 1 141
5. Subtract ten thousands: 7 – 4 = 3. Write 3 in the ten thousands place.	867 877 842 − 436 846 701 = _ _ _ _ 31 141
6. Subtract hundred thousands: 8 – 8 = 0. Write 0 in the hundred thousands place.	867 877 842 − 436 846 701 = _ _ _ 031 141
7. Subtract millions: 7 – 6 = 1. Write 1 in the millions place.	867 877 842 − 436 846 701 = _ _ 1 031 141
8. Subtract ten millions: 6 – 3 = 3. Write 3 in the ten millions place.	867 877 842 − 436 846 701 = _ 31 031 141
9. Subtract hundred millions: 8 – 4 = 4. Write 4 in the hundred millions place.	867 877 842 − 436 846 701 = 431 031 141
Therefore, 867 877 842 − 436 846 701 = 431 031 141.	

Subtraction of whole numbers arranged vertically without regrouping

Example

$$\begin{array}{r} 675\ 984\ 234 \\ -\ 123\ 564\ 012 \\ \hline \\ \hline \end{array}$$

Solution

The steps for vertical subtraction are the same as horizontal subtraction.
Start to subtract from ones to hundred millions.

$$\begin{array}{r} 6\ 7\ 5\ \ 9\ 8\ 4\ \ 2\ 3\ 4 \\ -\ 1\ 2\ 3\ \ 5\ 6\ 4\ \ 0\ 1\ 2 \\ \hline 5\ 5\ 2\ \ 4\ 2\ 0\ \ 2\ 2\ 2 \\ \hline \end{array}$$

Therefore, the answer is 552 420 222.

Exercise 5

Answer the following questions:

1. 556 768 867 − 42 560 340 =

2. 446 198 667 − 32 060 440 =

3. 643 476 482 − 231 354 251 =

4. 112 187 865 − 182 660 =

5. 692 357 678 − 581 024 243 =

6. $\begin{array}{r} 4\ 7\ 3\ \ 5\ 8\ 8\ \ 2\ 6\ 1 \\ -\ 3\ 2\ 1\ \ 3\ 4\ 4\ \ 2\ 4\ 0 \\ \hline \\ \hline \end{array}$

7. $\begin{array}{r} 895\ 456\ 261 \\ -\ 521\ 334\ 240 \\ \hline \\ \hline \end{array}$

8. $\begin{array}{r} 776\ 474\ 164 \\ -\ 446\ 244\ \ 023 \\ \hline \\ \hline \end{array}$

9. $\begin{array}{r} 765\ 897\ 111 \\ -\ 321\ 234\ 100 \\ \hline \\ \hline \end{array}$

10. 423 548 299
− 421 344 040

11. 43 416 422 − 1 304 201 =

12. 534 674 430 − 231 354 230 =

13. 773 650 645 − 31 350 221 =

14. 629 573 898 − 501 041 253 =

15. 611 333 666 − 501 000 456 =

16. 768 566 260
− 467 233 140

17. 234 456 876
− 111 444 240

18. 999 888 777
− 666 333 222

19. 700 500 200
− 500 000 200

20. 403 508 201
− 301 300 200

Subtraction of whole numbers arranged horizontally by regrouping

Example

867 352 140 − 295 771 671 =

Steps

1. Subtract ones: 0 – 1: It is not sufficient. Take 1 group of tens from 4 tens and regroup it to get 10 ones. Add ones: 10 + 0 = 10. Now, subtract ones: 10 – 1 = 9. Write 9 in the ones place. Remember, 1 tens was regrouped into 10 ones. Thus, 3 tens remained in the tens place.	867 352 140 − 295 771 671 = _ _ _ _ _ _ _ _9

2. Subtract tens: 3 – 7: It is not sufficient. Take 1 group of hundreds from 1 hundreds and regroup it to get 10 tens. Add tens: 10 + 3 = 13. Now, subtract tens: 13 – 7 = 6. Write 6 in the tens place. Remember 1 hundreds was regrouped into 10 tens. Thus, 0 hundreds remained in the hundreds place.	867 352 140 − 295 771 671 = _ _ _ _ _ _ _ 69
3. Subtract hundreds: 0 – 6: It is not sufficient. Take 1 group of thousands from 2 thousands and regroup it to get 10 hundreds. Add hundreds: 10 + 0 = 10. Now, subtract hundreds: 10 – 6 = 4. Write 4 in the hundreds place. Remember 1 thousands was regrouped into 10 hundreds. Thus, 1 thousands remained in the thousands place.	867 352 140 − 295 771 671 = _ _ _ _ _ _ 469
4. Subtract thousands: 1 – 1 = 0. Write 0 in the thousands place.	867 352 140 − 295 771 671 = _ _ _ _ _ 0 469
5. Subtract ten thousands: 5 – 7: It is not sufficient. Take 1 group of hundred thousands from 3 hundred thousands and regroup it to get 10 ten thousands. Add ten thousands: 10 + 5 = 15. Now, subtract ten thousands: 15 – 7 = 8. Write 8 in the ten thousands place. Remember 1 hundred thousands was regrouped into 10 ten thousands. Thus, 2 hundred thousands remained in the hundred thousands place.	867 352 140 − 295 771 671 = _ _ _ _ 80 469

6. Subtract hundred thousands: 2 – 7: It is not sufficient. Take 1 group of millions from 7 millions and regroup it to get 10 hundred thousands. Add hundred thousands: 10 + 2 = 12. Now, subtract hundred thousands: 12 – 7 = 5. Write 5 in the hundred thousands place. Remember 1 millions was regrouped into 10 hundred thousands. Thus, 6 millions remained in the millions place.	867 352 140 − 295 771 671 =_ _ _ 580 469
7. Subtract millions: 6 – 5 = 1. Write 1 in the millions place.	867 352 140 − 295 771 671 =_ _ 1 580 469
8. Subtract ten millions: 6 – 9: It is not sufficient. Take 1 hundred millions from 8 hundred millions and regroup it into 10 ten millions. Add ten millions: 10 + 6 = 16. Now, subtract ten millions: 16 – 9 = 7. Write 7 in the ten millions place. Remember, 1 hundred millions was regrouped into 10 ten millions. Thus, 7 hundred millions remained in the hundred millions place.	867 352 140 − 295 771 671 =_ 71 580 469
9. Subtract hundred millions: 7 – 2 = 5. Write 5 in the hundred millions place.	867 352 140 − 295 771 671 =571 580 469
Therefore, 867 352 140 – 295 771 671 = 571 580 469.	

Subtraction of whole numbers arranged vertically by regrouping

Example

476 647 821
− 259 465 160
_____

Steps

1. Subtract ones: 1 – 0. Write 1 in the ones place.	476 647 821 − 259 465 160 1
2. Subtract tens: 2 – 6: It is not sufficient. Take 1 group of hundreds from 8 hundreds and regroup it to get 10 tens. Add tens: 10 + 2 = 12. Now, subtract tens: 12 – 6 = 6. Write 6 in the tens place. Remember 1 hundreds was regrouped into 10 tens. Thus, 7 hundreds remained in the hundreds place.	7 12 476 647 821 − 259 465 160 61
3. Subtract hundreds: 7 – 1 = 6. Write 6 in the hundreds place.	7 12 476 647 821 − 259 465 160 661
4. Subtract thousands: 7 – 5 = 2. Write 2 in the thousands place.	7 12 476 647 821 − 259 465 160 2 661
5. Subtract ten thousands: 4 – 6: It is not sufficient. Take 1 group of hundred thousands from 6 hundred thousands and regroup it to get 10 ten thousands. Add ten thousands: 10 + 4 = 14. Now, subtract ten thousands: 14 – 6 = 8. Write 8 in the ten thousands place. Remember 1 hundred thousands was regrouped into 10 ten thousands. Thus, 5 hundred thousands remained in the hundred thousands place.	5 14 7 12 476 647 821 − 259 465 160 82 661
6. Subtract hundred thousands: 5 – 4 = 1. Write 1 in the hundred thousands place.	5 14 7 12 476 647 821 − 259 465 160 182 661

7. Subtract millions: 6 – 9: It is not sufficient. Take 1 group of ten millions from 7 ten millions and regroup it to get 10 millions. Add millions: 10 + 6 = 16. Now, subtract millions: 16 – 9 = 7. Write 7 in the millions place. Remember 1 ten millions was regrouped into 10 millions. Thus, 6 ten millions remained in the ten millions place.	6 16 5 14 7 12 476 647 821 − 259 465 160 7 182 661
8. Subtract ten millions: 6 – 5 = 1. Write 1 in the ten millions place.	6 16 5 14 7 12 476 647 821 − 259 465 160 17 182 661
9. Subtract hundred millions: 4 – 2 = 2. Write 2 in the hundred millions place.	6 16 5 14 7 12 476 647 821 − 259 465 160 217 182 661
Therefore, 476 647 821 − 259 465 160 = 217 182 661.	

Exercise 6

Answer the following questions:

1. 477 678 921 − 320 969 796 =
2. 631 200 349 − 579 999 291 =
3. 214 431 516 − 119 229 593 =
4. 241 341 641 − 188 477 980 =
5. 555 666 777 − 399 211 451 =
6. 679 796 211 − 498 289 999
7. 666 333 222 − 388 763 987

8. 587 444 200
− 298 211 900

9. 111 111 222
− 109 999 999

10. 981 111 111
− 789 000 999

11. 234 456 678 − 169 393 523 =

12. 123 890 790 − 20 999 299 =

13. 897 678 564 − 697 788 364 =

14. 761 167 333 − 261 967 333 =

15. 999 222 000 − 129 888 199 =

16. 334 559 881
− 234 670 000

17. 443 555 555
− 229 639 789

18. 882 288 288
− 875 189 299

19. 229 119 218
− 110 999 798

20. 513 413 313
− 492 199 313

Word problems on subtraction

Example 1

The government warehouse contains 9 581 867 bags of maize. If 7 560 656 bags of maize were distributed to needy citizens, how many bags of maize remained?

Solution

Total number of bags in the warehouse = 9 581 867
Number of bags distributed = 7 560 656
In this example, you can subtract vertically or horizontally to get the number of maize bags that remained in the warehouse. However, subtracting vertically is simpler. That is,

$$\begin{array}{rr} & 9\ 581\ 867 \\ - & 7\ 560\ 656 \\ \hline & 2\ 021\ 211 \\ \hline \end{array}$$

Therefore, 2 021 211 bags of maize remained in the warehouse.

Example 2

A certain region has 15 939 500 cattle. If another region has 8 604 563 cattle, find the difference between the number of cattle in the regions.

Solution

The number of cattle in the first region = 15 939 500
The number of cattle in the second region = 8 604 563
Subtract the numbers of cattle in the second region from the number of cattle in the first region.

That is,

$$\begin{array}{rr} & 15\ 939\ 500 \\ - & 8\ 604\ 563 \\ \hline & 7\ 334\ 937 \\ \hline \end{array}$$

Therefore, the difference between the number of cattle in both regions is 7 334 937.

Exercise 7

Answer the following questions:

1. The villagers harvested 16 659 450 kilograms of cotton. If 13 101 945 kilograms of the harvested cotton were clean, how many kilograms of cotton were not clean?

2. The poultry farmers in a certain region had 17 567 879 chickens. If 4 345 760 chickens died of a disease, how many chickens did the poultry farmers in that region remain with?

3. A school earned an income of 11 997 950 shillings. If the school spent 2 646 250 shillings for buying school equipment, how much money remained?

4. Four residential plots were sold for 84 500 000 shillings. If three plots were sold for 56 860 000 shillings, how much money did it cost for the fourth plot?

5. A textile industry manufactured 15 798 826 dresses for sale. If 3 693 420 dresses were defective, how many dresses were not defective?

6. A certain district has a total of 12 624 418 livestock. Among these livestock, 5 511 214 are cattle. What is the number of other livestock in that district?

7. A food crop company collected 12 829 068 kilograms of maize. If 9 708 032 kilograms were sold, how many kilograms remained?

8. Pupils made 11 879 325 bricks for the school project in one year. If 9 904 235 bricks were sold, how many bricks remained?

9. A factory manufactured 10 193 768 pairs of vitenge. If the factory sold 7 172 646 pairs of vitenge, how many pairs of vitenge were not sold?

10. A businessperson wanted to buy a car worth 18 000 000 shillings. He made a first payment of 10 550 150 shillings, how much money remained to complete the payment for that car?

11. Two regions planted 2 164 348 trees in various areas. If 1 998 786 trees grew well, how many trees did not grow well?

12. A school makes an annual profit of 1 530 450 shillings from a poultry project. If 765 980 shillings is used to buy chicks per year, how much money does the school remain with?

13. Water tanks are filled up with 3 675 987 litres of water per day. The daily use of water is 1 200 964 litres, how many litres of water remain in the tanks everyday?

14. A mason was paid 1 370 660 shillings as building cost for classrooms. If he used 498 700 shillings from that money to pay his day workers, how much money did he remain with?

15. Robert paid 1 460 000 shillings as house rent per year. He also paid 345 765 shillings as electricity bills per year. Find the difference between house rent and electricity bills per year.

16. In some regions in this country, about 2 870 664 livestock were branded. If the number of livestock in these regions is 3 234 861, how many livestock were not branded?

17. A sports betting club invited 1 121 612 sports fans to predict the football results. Among these fans, 498 678 were women. How many fans were men?

18. Eva's monthly salary is 2 485 300 shillings. She spends 430 800 shillings of her salary to pay house rent and 1 295 600 shillings to pay school fees. How much money does she remain with?

19. The number of people vaccinated in two regions was 1 684 500. If 847 312 of them were men, how many women were vaccinated?

20. A cement factory produced 3 984 784 bags of cement for business purposes. If 1 324 723 bags of cement were exported, how many bags of cement remained?

Revision exercise

Answer the following questions:

1. 328 246 957 + 320 969 796 =

2. 631 200 349 − 579 999 291 =

3. 639 482 522 − 356 269 796 =

4. 701 678 921 + 220 369 296 =

5. 337 278 821 + 520 939 796 =

6. 735 116 318 − 110 999 798 =

7. 577 878 921 − 333 222 346 =

8. 723 256 621 − 220 449 596 =

9. 888 111 421 + 12 399 196 =

10. 490 660 121 + 120 132 342 =

11. 488 668 922 − 330 669 346 =

12.
```
  239 519 628
+ 190 259 728
-------------
```

13.
```
  209 119 218
+ 710 999 798
-------------
```

14.
```
  249 119 218
+ 600 259 238
-------------
```

15.
```
  339 459 268
− 179 922 728
-------------
```

16.
```
  629 302 418
− 342 444 555
-------------
```

17.
```
  429 159 268
+  98 999 798
-------------
```

18.
```
  729 619 318
−  99 999 798
-------------
```

19.
```
  669 449 244
+  88 399 598
-------------
```

20.
```
  229 119 218
− 110 999 798
-------------
```

21. A country used petrol for three consecutive years as follows: 340 240 400 litres in 2013, 234 453 000 litres in 2014, and 370 220 150 litres in 2015. How many litres of petrol were used by that country in the three years?

22. A plot costs 40 500 000 shillings. The cost of building a house is 250 300 000 shillings. How much money will it cost for buying the plot and building the house altogether?

23. A certain region received 573 856 200 shillings for a development project. Another region received 380 675 400 shillings for the development project. How much money did the two regions receive?

24. Two institutions prepared promotion brochures as follows: The first institution prepared 456 322 300 brochures and the other institution prepared 340 202 000 brochures. How many brochures were prepared by the two institutions?

25. Which number when added to 657 498 235 gives the answer of 895 789 564?

26. A businessperson sold minerals and earned 11 950 000 shillings in one month. If 6 163 200 shillings was used as business cost, how much profit did the businessperson earn?

27. A clothier set aside 19 902 000 shillings for buying clothes. The clothier was given a discount and spent only 17 000 000 shillings. How much money did the clothier remain with?

28. Mr. Mtani borrowed 35 730 000 shillings from a bank and agreed to pay back 2 200 000 shillings each month. How much money does he owe the bank after the first month payment?

29. An amount of 17 816 450 shillings was set aside in a certain hardware store for ordering bags of cement from a factory. If the ordered cement costs 15 250 950 shillings, how much money remained?

30. Find the difference between 789 999 345 and 543 234 568.

Summary

1. When adding numbers, consider the place value of each digit in the given numbers. Begin by adding the ones, tens, hundreds, and so on up to the highest place value of the digit of the numbers you are adding.
2. You can add two or more numbers, horizontally or vertically.
3. When subtracting numbers, consider the place value of each digit in the given numbers. Begin by subtracting the ones, tens, hundreds, and so on up to the highest place value of the digit of the numbers you are subtracting.
4. You can subtract numbers horizontally or vertically.
5. When solving word problems involving addition or subtraction:
   (a) Interpret the explanation in order to decide on the operations to be used.
   (b) Word problems have the key information and explanation of the question.
   (c) Addition means finding the sum and subtraction means finding the difference of the given numbers.
   (d) After solving the problem, refer to the question, and then conclude by providing the correct answer.

Chapter Three

Multiplication of whole numbers

Introduction

In Standard Six, you learnt multiplication of whole numbers with a product not exceeding ten million (10 000 000). You also learnt the simple ways of multiplying numbers with more than one digit by aligning them vertically considering their place values. In this chapter, you will continue to learn about multiplication of whole numbers with a product which does not exceed one billion (1 000 000 000). You will also solve word problems on multiplication of whole numbers. The competencies gained will help you to solve a variety of real life problems involving whole numbers, such as running a business, preparing statistical data, estimating building materials, making financial transactions, and preparing budgets.

Exercise 1: Revision

Answer the following questions:

1. $865 \times 703 =$

2. $5\ 355 \times 831 =$

3. $1\ 387 \times 245 =$

4. $9\ 215 \times 341 =$

5. $637 \times 4\ 181 =$

6. $792 \times 6\ 563 =$

7.
$$\begin{array}{r} 4\ 3\ 0\ 0 \\ \times\quad 4\ 4\ 2 \\ \hline \\ \hline \end{array}$$

8.
$$\begin{array}{r} 8\ 1\ 4\ 1 \\ \times\quad 5\ 1\ 7 \\ \hline \\ \hline \end{array}$$

9.
$$\begin{array}{r} 1\ 0\ 5\ 9 \\ \times\ 6\ 2\ 1\ 5 \\ \hline \\ \hline \end{array}$$

10.
$$\begin{array}{r} 2\,1\,9\,0 \\ \times\quad 9\,5\,4 \\ \hline \\ \hline \end{array}$$

11.
$$\begin{array}{r} 3\,6\,0\,4 \\ \times\,1\,8\,8\,2 \\ \hline \\ \hline \end{array}$$

12.
$$\begin{array}{r} 4\,2\,8\,4 \\ \times\quad 7\,3\,9 \\ \hline \\ \hline \end{array}$$

13.
$$\begin{array}{r} 9\,3\,4\,6 \\ \times\quad 4\,9\,2 \\ \hline \\ \hline \end{array}$$

14.
$$\begin{array}{r} 7\,2\,8\,6 \\ \times\quad 6\,9\,2 \\ \hline \\ \hline \end{array}$$

15.
$$\begin{array}{r} 8\,3\,8\,6 \\ \times\quad 6\,9\,2 \\ \hline \\ \hline \end{array}$$

16.
$$\begin{array}{r} 9\,6\,4\,6 \\ \times\quad 5\,6\,9 \\ \hline \\ \hline \end{array}$$

17.
$$\begin{array}{r} 3\,5\,1\,3 \\ \times\quad 9\,6\,0 \\ \hline \\ \hline \end{array}$$

18.
$$\begin{array}{r} 5\,2\,6\,4 \\ \times\quad 6\,2\,8 \\ \hline \\ \hline \end{array}$$

19.
$$\begin{array}{r} 8\,4\,8\,5 \\ \times\quad 6\,9\,2 \\ \hline \\ \hline \end{array}$$

20.
$$\begin{array}{r} 7\,3\,2\,5 \\ \times\quad 6\,4\,3 \\ \hline \\ \hline \end{array}$$

21. A residential plot costs 2 751 250 shillings. What is the cost of four plots of the same size?

22. In a certain village, 1 650 villagers decided to contribute 5 300 shillings each for a water project. If all the villagers contributed, how much money was collected?

23. A businessperson bought 3 841 pawpaws. If each pawpaw was sold for 2 200 shillings, how much money did the businessperson spend to buy the pawpaws?

24. A construction company has 312 day workers. If each worker is paid 9 500 shillings per day, how much money does the company pay every day?

25. A dealer in building materials generates a profit of 350 750 shillings per day. How much profit will the dealer generate in a month with 31 days?

26. Ester buys 2 litres of milk every day. If one litre costs 2 150 shillings, how much money does she spend on buying milk in a year consisting of 365 days?

27. A bus carries 60 passengers. If each passenger pays a fare of 45 000 shillings, how much money does the bus owner collect in one trip?

28. A matchbox has 85 sticks. How many sticks are there in 14 250 matchboxes of the same type?

29. Clement sells 63 chickens every month. If the price of one chicken is 12 000 Shillings, how much money does he earn in a year?

30. In a certain African country, an airport can provide services to 462 015 passengers per month. If the number of passengers remains the same each month, how many passengers will be served by the airport in five months?

Multiplication of whole numbers with a product not exceeding 1 000 000 000

Example 1

284 865 × 2 016 =

Steps

1. Align numbers vertically by considering the place value of each digit, that is, ones in ones place, tens in tens place, hundreds in hundreds place and so on, up to the highest place value of the given number.
2. You can multiply the number by starting with the lowest place value (ones) of the multiplier towards the left side. You can also start with the highest place value of the multiplier towards the right side.
3. In this example, the multiplier is 2 016 and the multiplicand is 284 865. Start multiplying by the digits with the lowest place value in the multiplier, which is 6 (ones).

			2	8	4	8	6	5
		×			2	0	1	6
		1	7	0	9	1	9	0
		2	8	4	8	6	5	
	0	0	0	0	0	0		
+	5	6	9	7	3	0		
	5	7	4	2	8	7	8	4	0

Multiply by ones: 284 865 × 6

Multiply by tens: 284 865 × 1

Multiply by hundreds: 284 865 × 0

Multiply by thousands: 284 865 × 2

Therefore, 284 865 × 2 016 = 574 287 840.

Example 2

324 493 × 3 075 =

Solution

In this example, the multiplier is 3 075. Start multiplying the digit with the highest place value in the multiplier, which is 3, in thousands.

Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
			3	2	4	4	9	3
		×			3	0	7	5
9	7	3	4	7	9			
	0	0	0	0	0	0		
	2	2	7	1	4	5	1	
+		1	6	2	2	4	6	5
9	9	7	8	1	5	9	7	5

Therefore, 324 493 × 3 075 = 997 815 975.

Example 3

34 106 × 25 322 =

Solution

$$
\begin{array}{rrrrrrrrr}
 & & & 3 & 4 & 1 & 0 & 6 \\
 & & \times & 2 & 5 & 3 & 2 & 2 \\
\hline
 & & & 6 & 8 & 2 & 1 & 2 \\
 & & 6 & 8 & 2 & 1 & 2 & \\
 & 1 & 0 & 2 & 3 & 1 & 8 & \\
1 & 7 & 0 & 5 & 3 & 0 & & \\
+\ 6 & 8 & 2 & 1 & 2 & & & \\
\hline
8 & 6 & 3 & 6 & 3 & 2 & 1 & 3 & 2
\end{array}
$$

Therefore, 34 106 × 25 322 = 863 632 132.

Multiplication using a box method

Multiplication can be done using a box method. The following steps are used in the box multiplication method:

1. Draw lines to create rows and columns by considering the digits of the given numbers.
2. Write the multiplicand horizontally in the first row with each digit in a separate box. Also, write the multiplier vertically in the last column. In Example 4, the multiplicand and multiplier are written in red colour.
3. The multiplier can be written vertically in the first or last column. In Example 4, the multiplier is written vertically in the last column.
4. Divide each box diagonally to get two identical right-angled triangles, one at the top of the other as seen in Example 4. The green lines have been used in dividing the boxes.
5. Multiply the multiplicand with each digit in the multiplier starting from right to left.
6. In each box, write tens in the upper triangle and ones in the lower triangle.

7. Add the numbers along the parallelograms, trapezia or triangles formed between the parallel lines starting from the lower most corner at right hand side.
8. Write the product sequentially starting from the left end side to the right end side, in sequence. In Example 4, the arrows show how the product is obtained.

Example 4

324 493 × 3 075 =

Solution

	3	2	4	4	9	3	
	0 / 9	0 / 6	1 / 2	1 / 2	2 / 7	0 / 9	3
9	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0 / 0	0
9	2 / 1	1 / 4	2 / 8	2 / 8	6 / 3	2 / 1	7
7	1 / 5	1 / 0	2 / 0	2 / 0	4 / 5	1 / 5	5
	8	1	5	9	7	5	

Therefore, 324 493 × 3 075 = 997 815 975.

Activity: Multiplication by using box method

Find the value of 33 316 × 23 028 using the box method. Write the multiplicand horizontally in the first row and multiplier vertically in the last column.

Exercise 2

Answer the following questions:

1. 395 112 × 263 =

2. 4 146 × 3 512 =

3. 845 976 × 562 =

4. 46 756 × 16 864 =

5. 34 116 × 5 512 =

6. 734 134 × 242 =

7. 44 234 × 5 132 =

8. 593 786 × 422 =

9. 54 106 × 4 503 =

10. 522 776 × 445 =

11.
$$\begin{array}{r} 5\,4\,6\,8\,9\,3 \\ \times\quad\quad 2\,7\,6 \\ \hline \\ \hline \end{array}$$

12.
$$\begin{array}{r} 2\,4\,3\,8\,4\,3 \\ \times\quad\quad 1\,2\,3\,6 \\ \hline \\ \hline \end{array}$$

13.
$$\begin{array}{r} 3\,4\,4\,8\,8\,3 \\ \times\quad\quad 2\,4\,6\,6 \\ \hline \\ \hline \end{array}$$

14.
$$\begin{array}{r} 3\,2\,3\,3\,9\,3 \\ \times\quad\quad 3\,0\,7\,3 \\ \hline \\ \hline \end{array}$$

15.
$$\begin{array}{r} 3\,4\,1\,0\,5 \\ \times\,2\,4\,0\,4\,7 \\ \hline \\ \hline \end{array}$$

16.
$$\begin{array}{r} 2\,2\,5\,4\,3 \\ \times\,2\,1\,3\,4\,2 \\ \hline \\ \hline \end{array}$$

17.
$$\begin{array}{r} 4\,4\,6\,5\,5\,3 \\ \times\quad\quad 2\,1\,5\,6 \\ \hline \\ \hline \end{array}$$

18.
$$\begin{array}{r} 5\,7\,3\,2\,1\,8 \\ \times\quad\quad 6\,0\,6 \\ \hline \\ \hline \end{array}$$

19.
$$\begin{array}{r} 4\,6\,8\,9\,3 \\ \times\,1\,1\,4\,4\,6 \\ \hline \\ \hline \end{array}$$

20.
$$\begin{array}{r} 7\,2\,3\,4\,5\,7 \\ \times\quad\quad 8\,5\,0 \\ \hline \\ \hline \end{array}$$

Word problems on multiplication

Example 1

Marina harvested 30 876 pineapples from her farm. She sold each pineapple at 1 592 shillings. How much money did she earn?

Solution

Number of pineapples = 30 876
The price of one pineapple = 1 592 shillings

In order to obtain the amount of money she earned, multiply the number of pineapples by the price of one pineapple, that is:

$$\begin{array}{rrrrrrr} & & 3 & 0 & 8 & 7 & 6 \\ & \times & & 1 & 5 & 9 & 2 \\ \hline & & 6 & 1 & 7 & 5 & 2 \\ & 2 & 7 & 7 & 8 & 8 & 4 \\ 1 & 5 & 4 & 3 & 8 & 0 & \\ + 3 & 0 & 8 & 7 & 6 & & \\ \hline 4 & 9 & 1 & 5 & 4 & 5 & 9 & 2 \end{array}$$

Therefore, Marina earned 49 154 592 shillings.

Example 2

A house was constructed by using 15 020 bricks. How many bricks will be used to construct 23 136 houses of the same kind?

Solution

Number of bricks for one house = 15 020
To obtain the number of bricks required to construct 23 136 houses, multiply the number of bricks needed to construct one house by the number of houses to be constructed.

$$
\begin{array}{rrrrrrrrr}
 & & & 1 & 5 & 0 & 2 & 0 \\
 & & \times & 2 & 3 & 1 & 3 & 6 \\
\hline
 & & & 9 & 0 & 1 & 2 & 0 \\
 & & 4 & 5 & 0 & 6 & 0 & \\
 & 1 & 5 & 0 & 2 & 0 & & \\
4 & 5 & 0 & 6 & 0 & & & \\
+\,3 & 0 & 0 & 4 & 0 & & & \\
\hline
3 & 4 & 7 & 5 & 0 & 2 & 7 & 2 & 0
\end{array}
$$

Therefore, 23 136 houses of the same kind will require 347 502 720 bricks.

Exercise 3

Answer the following questions:

1. In a certain region, residents use 698 752 litres of water per hour. How many litres of water will be used by the residents in 1 286 hours?

2. An entrepreneurship training centre has 7 657 students. Each student pays 2 250 shillings per week. How much money does the centre collect per week?

3. One sack of rice costs 250 050 shillings. The farmers' association sold 259 sacks of rice at the same price. How much money did the association earn?

4. An exercise book has a mass of 850 grams. A donor bought 35 675 exercise books for an orphanage. What is the total weight of all the exercise books?

5. A car travels 75 649 metres per hour. How many metres will it travel in 315 hours?

6. In 2017, a one-way flight ticket by Air Tanzania from Dar es Salaam to Mwanza was 113 000 shillings. How much money did the airline collect from 165 passengers?

7. Every month Rajab saves 1 880 000 shillings from his business profit. What will be his total savings in four years?

8. Magdalena intends to start a hotel business. She wants to buy 72 876 plates. If each plate costs 1 850 shillings, find the amount of money which she will spend on buying the plates.

9. A tray of chicken eggs costs 7 500 shillings. If a poultry farmer sells 4 595 trays of eggs, how much money will the farmer get?

10. A certain factory has 1 537 day-workers. If each day-worker is paid 352 500 shillings per month, how much does the factory pay to the day-workers every month?

11. A bakery sells 56 789 breads every week. If each bread is sold at 1 550 shillings, find the total amount of money the bakery earns every week.

12. Rehema saves 154 250 shillings every month in order to buy a farm. What will be her savings in 5 years and 8 months?

13. The government provided tree seedlings to 243 villages for reforestation. If each village got 3 505 350 tree seedlings, how many tree seedlings did the government distribute to all villages?

14. A tourist hotel accommodates 243 tourists per day. Each tourist pays 105 550 shillings. How much money does the hotel earn in 28 days?

15. A pair of kanga costs 5 500 shillings. Find the amount of money needed to buy 2 675 pairs of such kanga.

16. A certain school uses 193 540 litres of water every month. How many litres of water will the school use in seven years?

17. A factory manufactures 13 458 bicycle tubes per day. How many bicycle tubes will the factory manufacture in 1 250 days?

18. A farmer can harvest 2 899 kilograms of maize in a year. If 8 099 farmers are capable of harvesting the same amount of maize per year, how many kilograms of maize will be harvested by all the farmers in a year?

19. A telecommunication company has a total of 18 332 registered subscribers. If each subscriber buys an airtime of 5 500 shillings in a day, how much money will the company earn from its subscribers in one day?

20. Find the product of 542 836 and 804.

Revision exercise

Answer the following questions:

1. 116 543 × 4 512 =

2. 34 716 × 24 562 =

3. 23 116 × 5 512 =

4. 288 106 × 2 512 =

5. 321 123 × 458 =

6.
$$\begin{array}{r} 244183 \\ \times\quad 2466 \\ \hline \\ \hline \end{array}$$

7.
$$\begin{array}{r} 43633 \\ \times\quad 224 \\ \hline \\ \hline \end{array}$$

8. 43 566 × 22 112 =

9. 3 181 × 198 512 =

10. 222 111 × 613 =

11. 443 396 × 811 =

12. 534 106 × 1 512 =

13.
$$\begin{array}{r} 9\,9\,6\,1\,1\,3 \\ \times\quad\quad 2\,1\,3 \\ \hline \\ \hline \end{array}$$

14.
$$\begin{array}{r} 7\,7\,8\,8\,8\,7 \\ \times\quad\quad 5\,7\,7 \\ \hline \\ \hline \end{array}$$

15.
$$\begin{array}{r} 6\,2\,5\,3\,3\,3 \\ \times\quad\quad 4\,6\,6 \\ \hline \\ \hline \end{array}$$

16.
$$\begin{array}{r} 7\,4\,4\,7\,7\,3 \\ \times\quad\quad 7\,4\,6 \\ \hline \\ \hline \end{array}$$

17.
$$\begin{array}{r} 5\,3\,8\,6\,3 \\ \times\quad 1\,4\,4\,6\,6 \\ \hline \\ \hline \end{array}$$

18.
$$\begin{array}{r} 9\,4\,4\,8\,8\,3 \\ \times\quad\quad 1\,4\,6 \\ \hline \\ \hline \end{array}$$

19.
$$\begin{array}{r} 3\,0\,4\,8\,3\,8 \\ \times\quad\quad 2\,6\,4\,6 \\ \hline \\ \hline \end{array}$$

20.
$$\begin{array}{r} 4\,4\,4\,8\,8\,8 \\ \times\quad\quad 3\,4\,6 \\ \hline \\ \hline \end{array}$$

21.
$$\begin{array}{r} 1\,9\,8\,3\,3\,3 \\ \times\quad\quad 5\,5\,5 \\ \hline \\ \hline \end{array}$$

22.
$$\begin{array}{r} 3\,2\,3\,1\,8\,3 \\ \times\quad\quad 2\,4\,7\,6 \\ \hline \\ \hline \end{array}$$

23.
$$\begin{array}{r} 3\,2\,1\,3\,4\,6 \\ \times\quad\quad 2\,7\,3\,9 \\ \hline \\ \hline \end{array}$$

24.
$$\begin{array}{r} 5\,6\,7\,6\,6\,6 \\ \times\quad\quad 1\,4\,6\,6 \\ \hline \\ \hline \end{array}$$

25.
$$\begin{array}{r} 1\,4\,4\,1\,8\,3 \\ \times\quad\quad 3\,4\,2\,2 \\ \hline \\ \hline \end{array}$$

26.
$$\begin{array}{r} 1\,2\,3\,1\,3\,3 \\ \times\quad\quad 4\,4\,2\,2 \\ \hline \\ \hline \end{array}$$

27. A cooperative union bought 3 295 chicken eggs from poultry farmers in different villages. Each farmer sold 8 250 eggs to the union. How many eggs did the union buy?

28. A construction company bought 4 220 bags of cement. Each bag of cement costs 13 500 shillings. How much money did the company spend on the cement?

29. The government provided a subsidy of fertiliser to 123 000 families. Each family received 94 kilograms of fertiliser. How many kilograms of fertiliser did the government distribute to the families?

30. An investor has a farm with a length of 9 564 metres and a width of 4 568 metres. Find the area of the investor's farm.

Summary

1. You can multiply numbers vertically or horizontally. However, the vertical multiplication is preferred for numbers with more than one digit.
2. You can multiply numbers starting from the right towards the left (from the digit with the lowest place value) or from the left towards the right (from the digit with the highest place value).
3. When solving word problems on multiplication, the following should be considered:
   (a) Interpret the information given in the question,
   (b) Obtain the numbers involved in the question, and
   (c) Multiply the numbers, and then answer the question based on its requirements.
4. You can multiply the numbers using the box method.

Chapter four

Division of whole numbers

Introduction

Steps for dividing whole numbers are the same as those you learnt from Standard Four to Standard Six. In this chapter, you will learn how to divide a number with a dividend not exceeding one billion (1 000 000 000) by a divisor not exceeding one hundred thousand (100 000). You will also solve word problems on division of whole numbers. The competencies gained will help you to apply numbers in business activities, buying and selling of commodities, making transactions, preparing statistical data, preparing budgets, and solving various challenges in daily life.

Exercise 1: Revision

Answer the following questions:

1. 1 321 897 ÷ 200 =
2. 324 531 ÷ 852 =
3. 694 500 ÷ 1 852 =
4. 6 410 625 ÷ 51 285 =
5. 5 434 878 ÷ 246 =
6. 1 433 385 ÷ 161 =
7. 130 704 ÷ 18 172 =
8. 472 708 ÷ 2 013 =
9. 3 527 228 ÷ 51 871 =
10. 66 306 ÷ 11 051 =
11. $138\overline{)62\,100}$
12. $123\overline{)43\,542}$
13. $350\overline{)8\,999\,900}$
14. $8\,526\overline{)3\,853\,752}$
15. $120\overline{)6\,693\,000}$
16. $150\overline{)7\,987\,050}$

17. A total of 648 645 crates of soda are to be carried by 120 lorries. If each lorry carries equal number of crates, how many crates will remain?

18. A lorry carried a total of 4 983 bags of cement in 11 trips. If it carried equal number of bags in each trip, how many bags of cement did the lorry carry in one trip?

19. A farmer earned 1 603 900 shillings after selling 746 jackfruits. What was the price of one jackfruit?

20. A cargo of 1 440 tons of sand was carried by 9 lorries of the same capacity. Each lorry has a capacity of carrying 20 tons per trip. If the lorries made equal number of trips, how many trips did each lorry make?

Dividing whole numbers with a dividend not exceeding 1 000 000 000

Steps for dividing whole numbers are the same as those you learnt in Standard Six. Division of large numbers becomes easier when the long division method is used.

Example 1

43 706 250 ÷ 225 =

Solution

Divide by using the long division method as follows:

```
          194250
225) 43706250
    –225
     2120
   – 2025
      956
    – 900
       562
     – 450
       1125
     – 1125
          0
        – 0
          0
```

Therefore, 43 706 250 ÷ 225 = 194 250.

Example 2

126 947 364 ÷ 93 619 =

Solution

Divide by using the long division method as follows:

```
          1356
93619) 126947364
     – 93619
       333283
     – 280857
        524266
      – 468095
         561714
       – 561714
                –
```

Therefore, 126 947 364 ÷ 93 619 = 1 356.

Example 3

24 463 239 ÷ 354 =

Solution

Divide by using the long division method as follows:

```
          69105
354) 24463239
    –2124
      3223
    – 3186
       372
     – 354
       183
     – 000
       1839
    –1770
       69
```

Therefore, 24 463 239 ÷ 354 = 69 105 remainder 69.

Exercise 2

Answer the following questions:

1. 112 045 640 ÷ 2 380 =
2. 440 220 880 ÷ 220 =
3. 230 439 440 ÷ 880 =
4. 666 333 666 ÷ 333 =
5. 29 230 450 ÷ 2 380 =
6. 890 290 280 ÷ 440 =
7. 46 231 830 ÷ 2 490 =
8. 93 439 360 ÷ 2 240 =
9. 449 339 270 ÷ 230 =
10. 694 652 112 ÷ 216 =
11. $220\overline{)82\,999\,900}$
12. $550\overline{)15\,555\,650}$
13. $250\overline{)35\,765\,500}$
14. $123\overline{)65\,888\,886}$
15. $357\overline{)28\,989\,828}$
16. $3500\overline{)123\,998\,000}$
17. $90\overline{)12\,009\,900}$
18. $200\overline{)10\,999\,000}$
19. $1250\overline{)882\,968\,750}$
20. $559\overline{)54\,999\,451}$

Word problems on division

Example 1

On average, 400 schools use a total of 12 666 800 litres of clean water per month. What is the average number of litres of clean water does each school use every month?

Solution

Total number of litres of water = 12 666 800 litres

Number of schools = 400

Divide the total number of litres of water by the number of schools as follows:

$$
\begin{array}{r}
31667 \\
400\overline{)\ 12666800} \\
-1200 \\
\hline
666 \\
-\ 400 \\
\hline
2668 \\
-\ 2400 \\
\hline
2680 \\
-\ 2400 \\
\hline
2800 \\
-\ 2800 \\
\hline
- \\
\end{array}
$$

Therefore, each school uses an average of 31 667 litres of clean water every month.

Example 2

During the construction of a community market, a contractor set aside 110 253 350 shillings for paying 152 day workers equally. How much money did each day worker get and how much balance did the contractor remain with?

Solution

Total amount of money = 110 253 350 shillings
Number of day workers = 152
Divide using long division method as follows:

$$
\begin{array}{r}
725350 \\
152\overline{)\ 110253350} \\
-1064 \\
\hline
385 \\
-\ 304 \\
\hline
813 \\
-\ 760 \\
\hline
533 \\
-\ 456 \\
\hline
775 \\
-\ 760 \\
\hline
150 \\
-\ 000 \\
\hline
150 \\
\end{array}
$$

Therefore, each day worker got 725 350 shillings and the contractor remained with 150 shillings.

Exercise 3

Answer the following questions:

1. A businessperson bought 34 water tanks of the same capacity for 62 738 500 shillings. What was the cost of each water tank?

2. A cooperative union earned a profit of 166 835 900 shillings after selling different types of crops. The money was equally shared among 6 454 members. How much money did each member get?

3. The cost of repairing a 65-kilometre road is 89 979 955 shillings. What is the cost of repairing a 1-kilometre road?

4. A certain village has 44 262 240 square metres of land. This land needs to be shared equally among 4 980 villagers for agricultural activities. How many square metres of land will each villager get?

5. The government supplied 210 454 720 ARV tablets to 3 760 HIV patients in a year. If the tablets were equally distributed among the patients, how many tablets did each patient receive?

6. A lorry uses an average of 1 litre of diesel to travel a distance of 4 kilometres. If it travels 346 848 kilometres, how many litres of diesel will it use?

7. Thom received a subsistence allowance of 36 879 780 shillings for his three years university studies. He was required to spend an equal amount of money in each year. How much money did he spend in each year?

8. A factory made 13 543 234 bricks. The bricks are to be shared equally among 145 schools for construction of laboratories. How many bricks will each school get and how many bricks will remain?

9. The villagers harvested 24 374 500 kilograms of beans. The beans were stored in large containers with a capacity of 15 300 kilograms each. How many kilograms of beans were not stored?

10. A factory made 11 437 sieves in a year. The sieves were sold for a total of 25 733 250 shillings. What was the price of each sieve?

11. A group of 12 business partners earned a profit of 667 056 000 shillings. If the profit was shared equally, how much money did each partner get?

12. A total of 16 232 176 empty sacks were equally distributed among 54 412 farmers. How many sacks did each farmer receive, and how many sacks remained?

13. A total of 45 445 000 iron sheets were bought for roofing 2 500 family houses. If each family received an equal number of iron sheets, how many iron sheets did each family receive?

14. A benefactors' union decided to donate 355 674 850 shillings to an orphanage. A total of 8 903 members of the union were required to contribute the donation equally. How much money did each benefactor contribute to meet the target?

15. An entrepreneur produces 222 553 750 millilitres of water. The water is filled in bottles with a capacity of 1 250 millilitres each. Find the number of bottles of the same capacity that are needed.

16. A bird flew a total distance of 268 379 016 metres in 366 days. If it flew the same distance each day, how long did it fly every day?

17. The government has a subsidy of 34 324 150 tons of fertiliser for farmers in 35 districts. If the fertiliser is distributed equally among the districts, how many tons of fertiliser will each district get?

18. A bank gained a profit of 931 385 734 shillings. The profit was shared equally among 934 bank shareholders. How much money did each shareholder get?

19. During an environmental conservation campaign, a total of 18 434 080 tree seedlings were planted. If 8 456 tree seedlings were planted in each row, how many rows were there?

20. In a certain region, a donor agency donated 987 456 050 shillings for renovating primary schools. The money was equally distributed among 350 schools. How much money did each school receive?

Revision exercise

Answer the following questions:

1. 494 123 616 ÷ 8 908 =
2. 654 183 072 ÷ 912 =
3. 49 443 950 ÷ 890 =
4. 154 873 270 ÷ 421 =
5. 500 900 600 ÷ 8 780 =
6. $250\overline{)307\ 650\ 500}$
7. $125\overline{)88\ 969\ 500}$
8. $2500\overline{)888\ 999\ 000}$
9. $1233\overline{)332\ 938\ 359}$
10. $127\overline{)822\ 229\ 115}$
11. $1276\overline{)112\ 918\ 344}$
12. $3350\overline{)442\ 468\ 000}$
13. $1766\overline{)662\ 668\ 542}$
14. 765 455 418 ÷ 7 098 =
15. 543 876 211 ÷ 1 321 =
16. 12 999 558 ÷ 654 =

17. 231 142 295 ÷ 8 777 =

18. 875 554 480 ÷ 6 544 =

19. $111\overline{)777\ 564\ 768}$

20. $12250\overline{)882\ 969\ 600}$

21. $187\overline{)882\ 949\ 859}$

22. $12750\overline{)83\ 257\ 500}$

23. $166\overline{)222\ 333\ 262}$

24. $555\overline{)672\ 558\ 990}$

25. $115\overline{)881\ 169\ 560}$

26. $110\overline{)801\ 099\ 970}$

27. The Medical Store Department received 675 472 678 malaria tablets from the Ministry of Health. The tablets were to be equally distributed among 5 674 health centres in the country. How many tablets did each health centre get?

28. A farmer bought 8 tractors for 267 750 000 shillings. If the price of the tractors was the same, how much money did the farmer spend on each tractor?

29. A government collected 897 803 865 shillings from vehicles tax levy. If the tax levy for each vehicle was 9 765 shillings, how many vehicles were taxed by the government?

30. A certain region has a total of 676 537 236 dairy cattle. The region has 6 987 villages each with the same number of dairy cattle. How many dairy cattle does each village have?

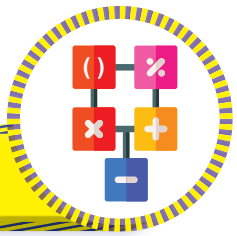
Summary

1. When dividing numbers, start from left by finding a number that is equal or greater than the divisor.
2. Consider the following when solving word problems on division:
   (a) Interpret the information given in the question,
   (b) Analyse the information to obtain the required numbers, and
   (c) Divide the numbers and obtain the required solution.

3. The number which you divide is called a dividend and the number which you divide by is called a divisor. The result obtained is called a quotient, and the number that is left after dividing is called a remainder.
4. When dividing whole numbers, you can get a quotient without a remainder or with a remainder.

Chapter Five

BODMAS



Introduction

Abbreviations and acronyms usually help us to easily remember various matters. They provide an easy way of recalling and presenting information whenever required. The long form of the acronym BODMAS is Brackets, Of, Division, Multiplication, Addition, and Subtraction. An expression can be correctly evaluated by following the order of operations of BODMAS. The competencies gained will help you to correctly evaluate and simplify mathematical expressions.

Order of operations of the BODMAS rule

The BODMAS rule explains the order of operations to be followed when evaluating or simplifying a mathematical expression. The first letters of words Brackets Of Division, Multiplication, Addition, and Subtraction are used to form the BODMAS rule as shown in the following table:

B	O	D	M	A	S
Bracket	Of	Division	Multiplication	Addition	Subtraction

Evaluation of an expression depends on the operations used in the given expression. An expression is evaluated or simplified by following the order of precedence of operations in the BODMAS rule. If an expression has two or more similar operations, the order of that operation will be from left to right.

The order of operations in accordance to priority given in the BODMAS rule when evaluating or simplifying an expression is as follows:

B represents Brackets
O represents Of
D represents Division
M represents Multiplication
A represents Addition
S represents Subtraction

Mathematical expressions involving two operations

When an expression involves only two operations, each operation is performed by following the order of operations given in the BODMAS rule.

Example 1

Find the value of $72 - 48 \div 3$.

Solution

Using BODMAS rule, follow the steps:

Start by dividing, that is divide 48 by 3:

$72 - 48 \div 3$

$= 72 - 16$

Finish by subtracting:

$72 - 16 = 56$

Therefore, $72 - 48 \div 3 = 56$.

Example 2

Find the value of $\frac{1}{6} + \frac{1}{2} \times \frac{1}{4}$.

Solution

Using BODMAS rule, follow the steps:

Start by multiplying, that is multiply $\frac{1}{2}$ by $\frac{1}{4}$:

$\frac{1}{6} + \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{6} + \frac{1}{8}$

Then add:

$\frac{1}{6} + \frac{1}{8} = \frac{4+3}{24}$

$= \frac{7}{24}$

Therefore, $\frac{1}{6} + \frac{1}{2} \times \frac{1}{4} = \frac{7}{24}$.

Example 3

Find the value of $2.65 + 0.33 - 1.14$.

Solution

Using the BODMAS rule, follow the steps:

Start by adding, that is add 2.65 and 0.33:

$2.65 + 0.33 - 1.14$

$= 2.98 - 1.14$

Finish by subtracting:

$2.98 - 1.14 = 1.84$

Therefore, $2.65 + 0.33 - 1.14 = 1.84$.

Exercise 1

Answer the following questions:

1. $6 + 7 \times 4 =$
2. $4 + 9 \div 3 =$
3. $20 + 9 - 7 =$
4. $72 \times 9 + 214 =$
5. $290 - 36 \times 4 =$
6. $\frac{1}{5} - \frac{1}{3} + \frac{1}{4} =$
7. $\frac{3}{8} \times 96 \div 4 =$
8. $142 \div 2 - \frac{1}{10} =$
9. $0.4 \times 15 + 18 =$
10. $6.5 \times 4.4 + 1.8 =$
11. $8 - 3 \times 2 =$
12. $13 - 12 \div 2 =$

13. $128 \div 4 + 58 =$

14. $7 \times 8 \div 4 =$

15. $336 \div 12 \times 18 =$

16. $\frac{1}{8} - \frac{1}{2} \times \frac{1}{5} =$

17. $\frac{2}{7} \times \frac{3}{5} + \frac{1}{6} =$

18. $30.25 + 4.75 - 28.35 =$

19. $38.5 \times 8 - 27.3 =$

20. $27.56 \times 32 \div 4 =$

Mathematical expressions involving three operations

An expression involving three operations is simplified or evaluated by performing each operation one after another by following the BODMAS rule.

Example 1

Find the value of $27 \times 2 - 4 + 15$.

Solution

Using the BODMAS rule, follow the steps:

Start by multiplying:

$27 \times 2 - 4 + 15$

$= 54 - 4 + 15$

Followed by adding positive integers:

$54 - 4 + 15 = 54 + 15 - 4$

$= 69 - 4$

Finally subtract:

$69 - 4 = 65$

Therefore, $27 \times 2 - 4 + 15 = 65$.

Example 2

Find the value of $320 \div 5 + \frac{4}{5} \times \frac{1}{9}$.

Solution

Using the BODMAS rule, follow the steps:

Start by dividing:

$320 \div 5 + \frac{4}{5} \times \frac{1}{9}$

$= 64 + \frac{4}{5} \times \frac{1}{9}$

Followed by multiplying:

$64 + \frac{4}{5} \times \frac{1}{9}$

$= 64 + \frac{4}{45}$

Finally add:

$64 + \frac{4}{45} = 64\frac{4}{45}$

Therefore, $320 \div 5 + \frac{4}{5} \times \frac{1}{9} = 64\frac{4}{45}$.

Example 3

Find the value of $180 \times \frac{4}{9} + 19.64 - 26.84$.

Solution

Using the BODMAS rule, follow the steps:

Start by multiplying:

$180 \times \frac{4}{9} + 19.64 - 26.84$

$= 80 + 19.64 - 26.84$

Followed by adding:

$80 + 19.64 - 26.84 = 99.64 - 26.84$

Finally subtract:

$99.64 - 26.84 = 72.8$

Therefore, $180 \times \frac{4}{9} + 19.64 - 26.84 = 72.8$.

Exercise 2

Answer the following questions:

1. $6 \times 8 \div 2 + 5 =$
2. $\frac{4}{7} \times 3 - 7 + 12.65 =$
3. $72 \div 8 + 16 \times 24 =$
4. $36 \div 6 \times 3 - 13 =$
5. $5 \times \frac{13}{40} - \frac{11}{80} + \frac{7}{160} =$
6. $\frac{1}{8} \times \frac{3}{5} + \frac{6}{7} - \frac{1}{4} =$
7. $144 \div 16 \times \frac{2}{9} - \frac{1}{16} =$
8. $1\frac{1}{2} \times 18 - 1\frac{1}{2} + 2\frac{1}{2} =$
9. $8.64 \times \frac{1}{4} - 4 + 16.64 =$
10. $45 - 23.96 + \frac{32}{35} \times 5.2 =$
11. $5 \times 3.21 + 4 - 7.65 =$
12. $12 + 8 - 4 \div 2 =$
13. $400 \div 8 \times \frac{19}{45} - 13.7 =$
14. $\frac{1}{5} \times \frac{8}{9} + \frac{1}{90} - \frac{13}{90} =$
15. $64.5 + 24.6 \times 0.5 - 36.8 =$
16. $16 + \frac{3}{5} - \frac{1}{10} \times 12.5 =$
17. $4416 \div 69 \times \frac{3}{128} + \frac{19}{64} =$
18. $2.25 \times 16 \div 4 - 0.96 =$
19. $16.48 \times \frac{1}{8} + 5\,580 \div 45 =$
20. $3\,612 \div 129 + \frac{17}{32} \times 8 =$

Mathematical expressions involving four operations

When an expression involves four operations, the evaluation or simplification is done by considering the order of priority given in the BODMAS rule.

Example 1

Find the value of $36 \div 9 + 48 - 10 \times 2$.

Solution

Using the BODMAS rule, follow the steps:

Start by dividing:

$36 \div 9 + 48 - 10 \times 2$

$= 4 + 48 - 10 \times 2$

Multiply:

$4 + 48 - 10 \times 2$

$= 4 + 48 - 20$

Add:

$4 + 48 - 20 = 52 - 20$

Finally subtract:

$52 - 20 = 32$

Therefore, $36 \div 9 + 48 - 10 \times 2 = 32$.

Example 2

Find the value of $6\frac{1}{4} \times 5\frac{1}{2} + 252 \div 7 - 1\frac{3}{5}$.

Solution

Using the BODMAS rule, follow the steps:

Start by dividing:

$6\frac{1}{4} \times 5\frac{1}{2} + 252 \div 7 - 1\frac{3}{5}$

$= 6\frac{1}{4} \times 5\frac{1}{2} + 36 - 1\frac{3}{5}$

Multiply:

$6\frac{1}{4} \times 5\frac{1}{2} + 36 - 1\frac{3}{5}$

$= \frac{25}{4} \times \frac{11}{2} + 36 - 1\frac{3}{5}$

$= \frac{275}{8} + 36 - 1\frac{3}{5}$

Add:

$\frac{275}{8} + 36 - 1\frac{3}{5}$

$= \frac{563}{8} - 1\frac{3}{5}$

Finally subtract:

$\frac{563}{8} - 1\frac{3}{5} = \frac{2\,751}{40}$

$= 68\frac{31}{40}$.

Therefore, $6\frac{1}{4} \times 5\frac{1}{2} + 252 \div 7 - 1\frac{3}{5} = 68\frac{31}{40}$.

Example 3

Find the value of $32.65 \times 8.4 - 1\,143 \div 127 + 51.64$.

Solution

Using the BODMAS rule, follow the steps:

Start by dividing:

$32.65 \times 8.4 - 1\,143 \div 127 + 51.64$

$= 32.65 \times 8.4 - 9 + 51.64$

Multiply:

$32.65 \times 8.4 - 9 + 51.64$

$= 274.26 - 9 + 51.64$

Add:

$274.26 - 9 + 51.\,64$

$= 274.26 + 51.64 - 9$

$= 325.9 - 9$

Finally subtract:

$325.9 - 9 = 316.9$

Therefore, $32.65 \times 8.4 - 1\,143 \div 127 + 51.64 = 316.9$.

Exercise 3

Answer the following questions:

1. $8 + 6 \times 3 - 18 \div 6 =$
2. $174 - \frac{13}{15} \times 46 + 4.2 \div \frac{2}{5} =$
3. $5.6 + 30 \div 5 \times 4 - 15 =$
4. $468 \div 13\frac{6}{13} \times 12.7 - 3.9 + 62 =$

5. $\frac{4}{9} + \frac{1}{18} \times 145 \div 29 - \frac{6}{7} =$

6. $2\frac{4}{5} + 1\frac{1}{8} + 1\,632 \div 68 - 2\frac{3}{4} =$

7. $16\frac{1}{2} - 12\frac{3}{4} \times \frac{1}{3} + 78 \div 13 =$

8. $19.44 \times 3.2 + 24 \div 2 - 6.21 =$

9. $221 \div 17 + 82 - 19 \times 5 =$

10. $46.38 - \frac{4}{9} + 22.5 \div 5.2 \times \frac{12}{19} =$

11. $9 \times 5 - 14 \times 6 \div 2 =$

12. $13 + 24 \div 4 \times 2 - 15 =$

13. $65 \div 13 \times \frac{27}{35} - 19 + 3.6 =$

14. $310 \times 6 + 52 \div 2 - 16 =$

15. $2.14 \times 0.15 + 57 \div 3 - 0.96 =$

16. $25.4 \times 6.5 + 39.8 - 56 \div 0.96 =$

17. $148 \div 3\frac{1}{4} + 6.9 \times \frac{5}{7} - 4.72 =$

18. $\frac{9}{11} \times 2\frac{1}{9} + 3\frac{1}{3} - 18 \div 6 =$

19. $\frac{16}{24} \times \frac{10}{25} - \frac{1}{5} + 90 \div 18 =$

20. $175.5 \times 10.2 + 9.8 - 24 \div 6 =$

Mathematical expressions involving operations and brackets

An expression consisting of brackets and all four basic mathematical operations which are division, multiplication, addition, and subtraction is evaluated or simplified by using a complete order of the BODMAS rule. Calculations are done as follows:

1. Open Brackets by solving the terms inside it. Multiply the number outside the bracket with the answer obtained.
2. Divide
3. Multiply
4. Add
5. Subtract

Activity: Constructing a monthly calendar using mathematical operations

Consider the following procedure for constructing a calendar for December this year:

1. Draw a rectangle on a manila paper using a marker pen.
2. On the rectangle, draw 6 vertical lines and 5 horizontal lines to make small rectangles.

3. Starting from Monday to Sunday, write names of the days from left to right in the first row.
4. Observe the last day of November.
5. Generate expressions by using mathematical operations on the year digits to fill in the empty rectangles.

   Example: In order to fill in the small rectangle of date 16, the expression $(9 - 1) \times (2 - 0) = 16$ will be made.

   Caution: A digit should be used only once in every expression.
6. Hang the calendar on the wall.

Example 1

Find the value of $3 \times 8 + (20 - 6) \div 2$.

Solution

Using the BODMAS rule, follow the steps:

Start by opening brackets:

$3 \times 8 + (20 - 6) \div 2 = 3 \times 8 + 14 \div 2$

Divide:

$3 \times 8 + 14 \div 2$

$= 3 \times 8 + 7$

Multiply:

$3 \times 8 + 7$

$= 24 + 7$

Finish by adding:

$24 + 7 = 31$.

Therefore, $3 \times 8 + (20 - 6) \div 2 = 31$.

Example 2

Find the value of $(63.6 + 36.4) \div \left(\frac{6}{7} - 8\right) + 58 \times \frac{2}{3} - 1\frac{1}{8}$.

Solution

Using the BODMAS rule, follow the steps:

Start by opening brackets:

$(63.6 + 36.4) \div \left(\frac{6}{7} - 8\right) + 58 \times \frac{2}{3} - 1\frac{1}{8}$

$= 100 \div \frac{-50}{7} + 58 \times \frac{2}{3} - 1\frac{1}{8}$

Divide:

$100 \div \frac{-50}{7} + 58 \times \frac{2}{3} - 1\frac{1}{8}$

$= -14 + 58 \times \frac{2}{3} - 1\frac{1}{8}$

Multiply:

$-14 + 58 \times \frac{2}{3} - 1\frac{1}{8}$

$= -14 + \frac{116}{3} - 1\frac{1}{8}$

Add:

$-14 + \frac{116}{3} - 1\frac{1}{8}$

$\frac{116}{3} - \frac{121}{8}$

Finally subtract:

$= \frac{116}{3} - \frac{121}{8} = \frac{565}{24}$

$= 23\frac{13}{24}$.

Therefore, $(63.6 + 36.4) \div \left(\frac{6}{7} - 8\right) + 58 \times \frac{2}{3} - 1\frac{1}{8} = 23\frac{13}{24}$.

Example 3

Find the value of

$(258.36 - 110.16) \div (61.49 - 24.44) + 0.38 \times 4.25 + 3.92$.

Solution

Using the BODMAS rule, follow the steps:

Start by opening brackets:

$(258.36 - 110.16) \div (61.49 - 24.44) + 0.38 \times 4.25 + 3.92$

$= 148.2 \div 37.05 + 0.38 \times 4.25 + 3.92$

Divide:
$148.2 \div 37.05 + 0.38 \times 4.25 + 3.92$
$= 4 + 0.38 \times 4.25 + 3.92$
Multiply:
$4 + 0.38 \times 4.25 + 3.92$
$= 4 + 1.615 + 3.92$
Finally add:
$4 + 1.615 + 3.92 = 9.535.$

Therefore,
$(258.36 - 110.16) \div (61.49 - 24.44) + 0.38 \times 4.25 + 3.92 = 9.535.$

Exercise 4

Answer the following questions:

1. $3 + 4(8 - 2) \div 6 =$
2. $5 + 2(14 + 36) \div 10 - (6 \div 3) + 328 =$
3. $2(5.6 - 2.1) + 3.3(8 + 3) \div 11 =$
4. $8 \times 3 + (5 + 6) - 15 \div 3 =$
5. $2(100 \div 5) + 6 \times 3 - 17 =$
6. $(28 \div 4) + 3 + (10 - 2) \times 5 =$
7. $4 \times 5 + (14 + 8) - 36 \div 9 =$
8. $14.2 - 8.6 + 3.4 + 8 \times (77.9 \div 8.2) =$
9. $24(120 - 95) + 29 \times 7 - 960 \div 120 =$
10. $(94 \times 7) \div 2 + (104 - 36) \times 4 - 16 + 32 =$
11. $\left(1\frac{9}{10} \times 95\right) + 29 \times 7 - 109.44 \div 9.12 =$

12. $\left(5\frac{7}{8} - 2\frac{3}{4}\right) \times \left(2\frac{5}{6} + 2\frac{1}{3}\right) + 216 \div 6 - 14 =$

13. $\frac{3}{7} \times 210 + \left(\frac{9}{14} - \frac{1}{7}\right) - 18 \div 3 + 72 =$

14. $0.4 \times 0.5 - \frac{3}{10} + 0.4 + \frac{1}{5}(8.4 \div 4) =$

15. $(75 \div 5) - \left(14\frac{4}{5} - 11\frac{1}{10}\right) + 5\frac{1}{7} \times 1\frac{3}{4} =$

16. $1.45 \times 1.8 + (64 - 52) \times 1.9 - 2.24 =$

17. $19.4 \times 6.15 - (18.2 - 5.2) + 256 \div 4 =$

18. $(25.65 + 12.35) \div (52.19 - 33.19) \times 0.18 =$

19. $(62 + 48) + 2\frac{1}{2} \times 3\frac{1}{5} - 2\frac{3}{8} + (48 \div 3) =$

20. $\left(\frac{3}{4} \text{ of } 2\frac{1}{6}\right) + \left(1\frac{2}{7} \text{ of } 1\frac{1}{3}\right) \times 250 \div 50 =$

Mathematical expressions on operations, brackets and exponents (powers)

These mathematical expressions are comprised of numbers, mathematical operations and exponents (powers). The acronym BEDMAS is used to express the order of priority of mathematical operations in a mathematical expression with exponents (powers). The following is the order of operations in BEDMAS:

B represents Brackets
E represents Exponents (powers)
D represents Division
M represents Multiplication
A represents Addition
S represents Subtraction

In calculating by using BEDMAS consider the following procedure:

1. Perform the operations in the brackets.
2. Calculate the exponents (powers).
3. Divide from left to right.
4. Multiply from left to right.
5. Add from left to right.
6. Lastly, subtract from left to right.

Example 1

Find the value of $(8 + 3)^2 + (25 - 13) \times 8 - 244 \div 4$.

Solution

Using the BEDMAS rule, follow the steps:

Start by opening brackets:

$(8 + 3)^2 + (25 - 13) \times 8 - 244 \div 4$

$= 11^2 + 12 \times 8 - 244 \div 4$

Find the square of 11:

$11^2 + 12 \times 8 - 244 \div 4$

$= 121 + 12 \times 8 - 244 \div 4$

Divide:

$121 + 12 \times 8 - 244 \div 4$

$= 121 + 12 \times 8 - 61$

Multiply:

$121 + 12 \times 8 - 61$

$= 121 + 96 - 61$

Add:

$121 + 96 - 61$

$= 217 - 61$

Finally subtract:

$217 - 61 = 156$.

Therefore, $(8 + 3)^2 + (25 - 13) \times 8 - 244 \div 4 = 156$.

Example 2

Find the value of $\left(\frac{2}{3} - \frac{3}{7}\right)^2 + 1\,984 \div 124 - 1\frac{1}{5} \times 4\frac{1}{2}$.

Solution

Using the BEDMAS rule, follow the steps:

Start by opening brackets:

$\left(\frac{2}{3} - \frac{3}{7}\right)^2 + 1\,984 \div 124 - 1\frac{1}{5} \times 4\frac{1}{2}$

$= \left(\frac{5}{21}\right)^2 + 1\,984 \div 124 - 1\frac{1}{5} \times 4\frac{1}{2}$

Find the square of $\frac{5}{21}$:

$\left(\frac{5}{21}\right)^2 + 1\,984 \div 124 - 1\frac{1}{5} \times 4\frac{1}{2}$

$= \frac{25}{441} + 1\,984 \div 124 - 1\frac{1}{5} \times 4\frac{1}{2}$

Divide:

$\frac{25}{441} + 1\,984 \div 124 - 1\frac{1}{5} \times 4\frac{1}{2}$

$= \frac{25}{441} + 16 - 1\frac{1}{5} \times 4\frac{1}{2}$

Multiply:

$\frac{25}{441} + 16 - 1\frac{1}{5} \times 4\frac{1}{2}$

$= \frac{25}{441} + 16 - \frac{27}{5}$

Add:

$\frac{25}{441} + 16 - \frac{27}{5} = \frac{7\,081}{441} - \frac{27}{5}$

Finally subtract:

$\frac{7\,081}{441} - \frac{27}{5} = \frac{23\,498}{2\,205}$

$= 10\frac{1\,448}{2\,205}$.

Therefore, $\left(\frac{2}{3} - \frac{3}{7}\right)^2 + 1\,984 \div 124 - 1\frac{1}{5} \times 4\frac{1}{2} = \frac{23\,498}{2\,205}$ or $10\frac{1\,448}{2\,205}$.

Example 3

Find the value of $12.45 \times 0.6 + (9.8 - 1.4)^2 - 164 \div 4 + 1.5^2$

Solution

Using the BEDMAS rule, follow the steps:

Start by opening brackets:

$12.45 \times 0.6 + (9.8 - 1.4)^2 - 164 \div 4 + 1.5^2$

$= 12.45 \times 0.6 + 8.4^2 - 164 \div 4 + 1.5^2$

Find the squares of 8.4 and 1.5:

$12.45 \times 0.6 + 8.4^2 - 164 \div 4 + 1.5^2$

$= 12.45 \times 0.6 + 70.56 - 164 \div 4 + 2.25$

Divide:
$12.45 \times 0.6 + 70.56 - 164 \div 4 + 2.25 = 12.45 \times 0.6 + 70.56 - 41 + 2.25$

Multiply:
$12.45 \times 0.6 + 70.56 - 41 + 2.25 = 7.47 + 70.56 - 41 + 2.25$

Add:
$7.47 + 70.56 - 41 + 2.25 = 7.47 + 70.56 + 2.25 - 41$
$= 80.28 - 41$

Finally, subtract:
$80.28 - 41 = 39.28.$

Therefore, $12.45 \times 0.6 + (9.8 - 1.4)^2 - 164 \div 4 + 1.5^2 = 39.28.$

Exercise 5

Answer the following questions:

1. $5^2 + (9 - 3) =$
2. $6 + (3 \times 4) \div 2^2 =$
3. $154 - (8 + 3)^2 \div 11 \times 13 =$
4. $(200 - 185)^2 \div 15 + 25 =$
5. $8^2 + (72 - 28) \div 4 =$
6. $(18 + 12)^2 - 25 \times 4 =$
7. $10^2 + (50 - 6) \div 4 =$
8. $72 - 6 \times 2 + (3 + 5)^2 =$
9. $92 + (8 - 5)^2 + 12 \times 2 =$
10. $33 - 5^2 + (170 + 80) \div 25 =$

11. $6\frac{1}{2} \times 1\frac{1}{13} + (56 \div 4)^2 - \left(7\frac{4}{7} \times \frac{3}{7}\right) =$

12. $(0.8 \times 4)^2 + 0.34 + 0.25 - 66 \div 11 =$

13. $12\frac{1}{6} \times 10\frac{1}{8} - \frac{897}{12} + \left(54 \div 6\right)^2 =$

14. $12 \times 0.6 + \left(2\frac{3}{5} - 1\frac{4}{5}\right)^2 + 12 \div \frac{1}{4} \times 0.1 =$

15. $24\frac{1}{8} + 2\frac{3}{8} - \frac{4}{13} \times \frac{5213}{16} + \left(2 \div \frac{1}{8}\right)^2 =$

16. $(120 - 97)^2 + (58 \div 2)^2 - 18 \times 5 - 17 =$

17. $2.45 + (1.85 - 0.15)^2 + 0.82 \times 3 =$

18. $26.8 + 1.4 - 1.7 \times 1.2 + (2.25 - 0.15)^2 =$

19. $1\frac{1}{3} + 4\frac{2}{3} - \left(5\frac{1}{3} \times \frac{3}{32}\right)^2 + 216 \div 18 =$

20. $64.6 - 71.4 - 6(1.8 \times 0.2)^2 + 120 \div 6 =$

Revision exercise

Answer the following questions:

1. Find the value of the following expressions:

(a) $3 + 2 \times 13 =$

(b) $(80 - 72) \div 4 + (28 - 16) =$

(c) $6 - 8 \div 2 =$

(d) $(3.24 \times 2.5) + (62 \div 2)^2 - 300 =$

(e) $\frac{4}{5} \times \frac{1}{3} - 2\left(7\frac{1}{2} \div 1\frac{1}{4}\right) =$

(f) $315 - 201 + 105 \times 4 \div 2 =$

(g) $(144 + 56) \div 5 + 9 \times (30 - 12)^2 =$

(h) $36 - (10 + 2)^2 \div 9 + 5 \times 4 =$

(i) $25 \times 2 - 42 \div 6 + 18 =$

(j) $\left(1\frac{1}{4} - \frac{1}{6}\right) + \frac{1}{5} - \left(\frac{2}{3} \times \frac{2}{5}\right) =$

2. Place the operation signs and/or brackets to make the computed answers correctly.

(a) 4 4 3 = 16

(b) 7 6 11 = 12

(c) 2 9 4 = 22

(d) 2 2 4 = 16

(e) 4 3 3 = −5

(f) 16.75 3.25 6.75 3.17 = 16.42

(g) 4 6 4 = 20

(h) $14\frac{2}{7}$ $\frac{5}{7}$ 60 11 = 64

(i) 2.5 1.2 2.6 0.6 = 1.7

(j) $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{24}$ = $\frac{13}{24}$

3. Calculate:

(a) $(7 + 5)^2 - 72 + 36 \div 3 =$

(b) $36 - 2(20 + 12 \div 4 \times 3 - 2 \times 2) + 20 =$

(c) $25.2 \times 5 + (0.3 + 0.9)^2 + 12 \div 6 =$

(d) $(18.4 - 12.3)^2 - (0.6 \times 0.4) + (12 \div 3)^2 =$

(e) $\left(\frac{4}{5} \div \frac{16}{5}\right) - \left(\frac{1}{2} - \frac{1}{4}\right)^2 + 81 \div 27 =$

(f) $60 \div (2 + 13) + 2(18 - 13)^2 =$

(g) $\frac{5}{8} + \frac{1}{4} + (16 \div 4)^2 \times \frac{3}{4} =$

(h) $2\frac{1}{3} \times \frac{1}{4} - 1\frac{1}{6} + (256 \div 64)^2 =$

(i) $(28 \div 4) + 3 + (10 - 8) \times 5 =$

(j) $20.5 \times 1.4 + (0.6 \times 0.5)^2 - 49 \div 7 =$

Summary

1. BODMAS is a mathematical rule formed by combining mathematical operations which are Brackets (B), Of (O), Division (D), Multiplication (M), Addition (A), and Subtraction (S).
2. If an expression has two or more similar operations, simplify the expression by doing the operations from left to right.
3. Electronic devices related to mathematics such as computers and calculators, are designed to perform mathematical operations by adhering to the BODMAS rule.
4. BEDMAS is an expression with mathematical operations, brackets, and exponents (powers). The arrangement expresses the priorities of mathematical operations in an expression with exponents (powers).

Chapter Six

Ratios

Introduction

Ratio is the relationship of two or more numbers or quantities by division. In this chapter, you will learn to identify and simplify ratios of two or more numbers or quantities and solve word problems on ratios. The competencies gained will help you to compare various quantities in daily life. Also, the study of ratios will enable you to calculate the dividends of various quantities under given proportions.

Ratio of two quantities

The ratio of two quantities shows how one quantity is compared to another quantity. The ratio of two quantities can be written in three ways which are:

(a) In words: For example, 2 cows to 3 goats. The word "to" represents the ratio of cows to goats.

(b) Using a colon (:): For example, 2 cows to 3 goats can be written as 2 : 3.

(c) In fractions: For example, 2 : 3 can be written as $\frac{2}{3}$.

Equivalent ratios

Pairs of ratios of two quantities can be compared. Sometimes, two ratios may seem different but are equivalent. Their equivalence can be observed when the ratios are simplified to their lowest terms.

For example, 2 : 3 = 8 : 12. The right-side ratio becomes 2 : 3 when simplified to its lowest term. Therefore, it is equivalent to the left-side ratio.

To simplify the ratios, the following steps are useful:

(a) Find the Greatest Common Factor (G.C.F) of the given numbers.

(b) Divide each number in a given ratio by the G.C.F of the numbers, and then simplify.

Example 1

A teacher's guide has 360 pages and a pupil's book has 480 pages.

(a) Write down the ratio of the number of pages in the teacher's guide to that of pupil's book.

(b) Find the ratio of the number of pages in the pupil's book to the total number of pages in the two books.

Solution

(a) Write the number of pages in the teacher's guide followed by the number of pages in the pupil's book separated by a colon (:). That is,

Teacher's guide : Pupil's book = 360 : 480

Find the G.C.F of the given numbers, and then divide each number by the G.C.F. In this case, the G.C.F of 360 and 480 is 120.

Thus, $\frac{360}{120} : \frac{480}{120} = 3 : 4$

Therefore, the ratio of the pages in the teacher's guide to the pupil's book is 3 : 4.

(b) Find the total number of pages in the two books:

$360 + 480 = 840$.

Using colon, separate the page numbers of the pupil's book from the total number of pages in the two books.

Start with the number of pages in the pupil's book, followed by the total number of pages in both books. That is,

Pupil's book: Total number of pages in both books = 480 : 840.

Find the G.C.F of the numbers involved, and then divide each number by the G.C.F. In this case, the G.C.F of 480 and 840 is 120.

Thus, $\frac{480}{120} : \frac{840}{120} = 4 : 7$.

Therefore, the ratio of the pages in the pupil's book to the total number of pages in the two books is 4 : 7.

Example 2

A class has 5 girls and 7 boys.
(a) Write the ratio of the number of girls to that of boys.
(b) Find the ratio of girls to the total number of pupils.

Solution

(a) Find the G.C.F of the numbers given, and then divide all numbers by the G.C.F of 5 and 7 which is 1.

Number of girls : Number of boys = 5 : 7
The G.C.F of 5 and 7 is 1

So, $\frac{5}{1} : \frac{7}{1} = 5 : 7.$

Therefore, the ratio of girls to boys is 5 : 7.

(b) The total number of pupils is $5 + 7 = 12$.
Therefore, the ratio of girls to the total number of pupils is 5 : 12.

Example 3

Simplify 16 : 28.

Solution

The GCF of 16 and 28 is 4.
Divide by G.C.F throughout:

$\frac{16}{4} : \frac{28}{4} = 4 : 7.$

Alternative solution 1

$16 : 28 = 4 \times 4 : 4 \times 7$

$= \frac{4 \times 4}{4} : \frac{4 \times 7}{4}$

$= 4 : 7.$

Alternative solution 2

Write the numbers in fraction. That is, $\frac{16}{28}$, and then simplify by using the common factors:

$\frac{16}{28} = \frac{8}{14} = \frac{4}{7}.$

Therefore, 16 : 28 = 4 : 7.

Example 4

Simplify 35 : 90.

Solution

The G.C.F of 35 and 90 is 5. Thus, divide each number by 5.

$\frac{35}{5} : \frac{90}{5}$

7 : 18.

Therefore, 35 : 90 = 7 : 18.

Example 5

Simplify 18 : 12.

Solution

The G.C.F of 18 and 12 is 6. Thus, divide each number by 6.

$\frac{18}{6} : \frac{12}{6} = 3 : 2.$

Therefore, 18 : 12 = 3 : 2.

Equivalent ratio table

The ratio of two quantities can be classified into groups of equivalent ratios and tabulated in the equivalent ratio table. With equivalent ratio table, one can find the missing numbers in some given ratios by using the following steps:

1. Inspect the table and identify a column which is complete, that is, it does not have an unknown. Compute the ratio of the two numbers in the column and use that ratio as a fixed reference for calculating the unknown by comparison.
2. Form a simple equation by comparing the ratio with the missing value to the fixed reference ratio.
3. Solve the simple equation to get the missing value in the table.

Example 1

Fill in the following equivalent ratio table by finding the unknown represented by the letters.

p	16	20	36	r
5	a	25	q	60

Solution

Follow the steps below:

1. Check the table to identify a column which is complete. In the given table, the third column is complete.
2. Write and simplify the ratio of the numbers in the third column by dividing each number by the G.C.F, which is 5.

   Thus, $\frac{20}{5} : \frac{25}{5} = 4 : 5$.

3. In order to find the value of ''p'', compare the first column ratio to the ratio of the third column as follows:

   $p : 5 = 4 : 5$, which is the same as $\frac{p}{5} = \frac{4}{5}$.

   Thus, $5p = 4 \times 5$. Divide by 5 both sides:

   $$\frac{5p}{5} = \frac{4 \times 5}{5}$$

   $$p = \frac{20}{5} = 4.$$

   Therefore, $p = 4$.

4. In order to find the value of ''a'', compare the second column ratio to that of the third column as follows:

   $16 : a = 4 : 5$, which is the same as $\frac{16}{a} = \frac{4}{5}$.

   Thus, $4a = 16 \times 5$. Divide by 4 both sides:

   $$\frac{4a}{4} = \frac{16 \times 5}{4}$$

   $$a = \frac{80}{4} = 20.$$

   Therefore, $a = 20$.

5. In order to find the value of ''q'', compare the fourth column ratio to the third column ratio as follows:
$36 : q = 4 : 5$, which is the same as $\frac{36}{q} = \frac{4}{5}$.

Thus, $4q = 36 \times 5$. Divide by 4 both sides:

$$\frac{4q}{4} = \frac{36 \times 5}{4}$$

$$q = \frac{180}{4} = 45.$$

Therefore, $q = 45$.

6. In order to find the value of ''r'', compare the fifth column ratio to the third column ratio as follows:
$r : 60 = 4 : 5$. This is the same as $\frac{r}{60} = \frac{4}{5}$.

Thus, $5r = 4 \times 60$. Divide by 5 both sides:

$$\frac{5r}{5} = \frac{4 \times 60}{5}$$

$$r = \frac{240}{5} = 48.$$

Therefore, $r = 48$.

Therefore, the missing numbers are bolded in the following table:

4	16	20	36	**48**
5	**20**	25	**45**	60

Example 2

Find the values of the variables in the following equivalent ratio table:

3	b	21	d	45	162	240
a	28	c	63	e	378	f

Solution

Follow the steps below:

1. Check the table to identify the column which is complete. In the given table, the sixth column is complete.
2. Simplify the ratio of numbers in the sixth column by dividing each number by the G.C.F, which is 54.

$$\frac{162}{54} : \frac{378}{54} = 3 : 7.$$

3. In order to find the value of ''a'', compare the ratio of the first column to that of the sixth column as follows:
   $3 : a = 3 : 7$, which is the same as $\frac{3}{a} = \frac{3}{7}$.

   So, $3a = 3 \times 7$. Divide by 3 both sides:

$$\frac{3a}{3} = \frac{3 \times 7}{3}$$

$$a = \frac{21}{3} = 7.$$

   Therefore, $a = 7$.

4. In order to find the value of ''b'', compare the second column ratio to the sixth column ratio as follows:
   $3 : 7 = b : 28$, which is the same as $\frac{3}{7} = \frac{b}{28}$.

   So, $7b = 3 \times 28$. Divide by 7 both sides:

$$\frac{7b}{7} = \frac{3 \times 28}{7}$$

$$b = \frac{84}{7} = 12.$$

   Therefore, $b = 12$.

5. In order to find the value of ''c'', compare the third column ratio to the sixth column ratio as follows:
3 : 7 = 21 : c, which is the same as $\frac{3}{7} = \frac{21}{c}$.

So, $3c = 7 \times 21$. Divide by 3 both sides:

$$\frac{3c}{3} = \frac{7 \times 21}{3}$$

$$c = \frac{147}{3} = 49.$$

Therefore, $c = 49$.

6. In order to find the value of ''d'', compare the fourth column ratio to the sixth column ratio as follows:
3 : 7 = d : 63, which is the same as $\frac{3}{7} = \frac{d}{63}$.

So, $7d = 3 \times 63$. Divide both sides by 7:

$$\frac{7d}{7} = \frac{3 \times 63}{7}$$

$$d = \frac{189}{7} = 27.$$

Therefore, $d = 27$.

7. In order to find the value of ''e'', compare the fifth column ratio to the sixth column ratio as follows:
3 : 7 = 45 : e, which is the same as $\frac{3}{7} = \frac{45}{e}$.

So, $3e = 7 \times 45$. Divide both sides by 3:

$$\frac{3e}{3} = \frac{7 \times 45}{3}$$

$$e = \frac{315}{3} = 105.$$

Therefore, $e = 105$.

8. In order to find the value of ''f'', compare the seventh column ratio to the sixth column ratio as follows:

$3 : 7 = 240 : f$, which is the same as $\frac{3}{7} = \frac{240}{f}$.

So, $3f = 7 \times 240$. Divide both sides by 3:

$$\frac{3f}{3} = \frac{7 \times 240}{3}$$

$$f = \frac{1680}{3} = 560.$$

Therefore, a = 7, b = 12, c = 49, d = 27, e = 105, f = 560.

Exercise 1

Answer the following questions:

1. Use a colon to write the following ratios:

(a) 3 to 7 (b) 5 to 11 (c) 7 to 9
(d) 13 to 19 (e) 4 to 13

2. Use a colon to write the following fractions into ratios:

(a) $\frac{9}{10}$ (b) $\frac{6}{13}$ (c) $\frac{11}{13}$ (d) $\frac{7}{19}$

3. Simplify the following ratios:

(a) 78 : 36 (b) 36 : 45 (c) 72 : 96
(d) 15 : 25 (e) 70 : 40

4. Find the value of x:

(a) $2 : 3 = 26 : x$ (b) $5 : 7 = (3x + 3) : 84$
(c) $(x - 1) : 3 = 16 : 24$ (d) $3 : 4 = 15 : x$

5. Fill in the empty spaces in the following table, if its columns have equivalent ratio:

18		30	42	
	12	15		30

6. Find the value of each letter in the following table, if its columns have equivalent ratio:

12	36	60	c	d
a	78	b	143	169

Finding the ratio of two quantities

The ratio of two quantities can be calculated using the following steps:

1. Express the given quantities A and B in ratio form, that is, A : B.
2. Add the given quantities, that is, A + B.
3. Write the ratio of the given quantities to the sum of all quantities in fraction, that is, $\frac{A}{A+B}$ and $\frac{B}{A+B}$.
4. If you are given another quantity, for example C, multiply each proportion by the given quantity C:

$$\frac{A}{A+B} \times C \text{ and } \frac{B}{A+B} \times C.$$

The answers in step 4 are the values of each quantity in the ratio of quantities A and B, respectively, with the given share C.

Activity: Finding the ratio of two quantities using a table

Follow the given instructions to do this activity using the following table of equivalent ratio:

Beans	1	6		15	30
Maize			30	45	

1. Prepare 100 and 200 grains of beans and maize respectively.
2. If the columns have an equivalent ratio, use the fourth completed column to arrange the grains of beans and maize in the given table.

Example 1

Two villages shared a revenue of 16 000 000 shillings in the ratio of 2 : 3. How much did each village get?

Solution

The ratio of the villages is as follows:
(First village) : (Second village) = 2 : 3
Add the ratios: 2 + 3 = 5.
The total revenue for all villages = 16 000 000 shillings
Divide the revenue as follows:

The first village: $\frac{2}{5} \times$ sh 16 000 000 = sh 6 400 000.

The second village: $\frac{3}{5} \times$ sh 16 000 000 = sh 9 600 000.

Therefore, the first village got 6 400 000 shillings and the second village got 9 600 000 shillings.

Example 2

Asha bought shares for 600 000 shillings in a cooperative shop. She divided the share to her two children, Juma and Halima in the ratio of 5 : 7. How much money did each child get?

Solution

Arrange the ratio of Juma to Halima as follows:
Juma : Halima = 5 : 7
Add the ratios: 5 + 7 = 12
The total share = 600 000 shillings

Calculate the amount of money that each child got as follows:

Juma: $\frac{5}{12} \times$ sh 600 000 = sh 250 000

Halima: $\frac{7}{12} \times$ sh 600 000 = sh 350 000.

Therefore, Juma got 250 000 shillings and Halima got 350 000 shillings.

Example 3

If A : B = 2 : 3, find the value of:
(a) B if A = 30
(b) A if B = 1 050

Solution

(a) Write A : B = 2 : 3 in fraction:

That is, $\frac{A}{B} = \frac{2}{3}$.

To find the value of B, substitute A = 30 in the above fraction:

$\frac{30}{B} = \frac{2}{3}$.

Cross-multiply as follows:

$2 \times B = 3 \times 30$

$2B = 90$.

Divide both sides by 2:

$\frac{2B}{2} = \frac{90}{2}$

$B = \frac{90}{2} = 45$.

Therefore, B = 45.

(b) Write A : B = 2 : 3 in fraction:

That is, $\frac{A}{B} = \frac{2}{3}$.

To find the value of A, substitute B = 1 050 in the above fraction:

$\frac{A}{1\,050} = \frac{2}{3}$.

Cross-multiply as follows:

$3 \times A = 2 \times 1\ 050$

$3A = 2\ 100.$

Divide both sides by 3:

$$\frac{3A}{3} = \frac{2\ 100}{3}$$

$$A = \frac{2\ 100}{3} = 700.$$

Therefore, $A = 700$.

Exercise 2

Answer the following questions:

1. If M : N = 12 : 21, find the value of N given M = 300.
2. If J : K = 80 : 120, find the value of J given K = 3.
3. If C : D = 28 : 336, find the value of D given C = 7.
4. If 4 : 9 = T : 324, find the value of T.
5. Zawadi mixed maize and wheat flour in the ratio of 9 : 7. If the mixture had a mass of 512 kilograms, calculate the mass of:
   (a) Maize flour.
   (b) Wheat flour.
6. Hawa has 72 tomato seedlings and Jonathan has 96 tomato seedlings. Find the ratio of their seedlings.
7. Getrude and Hassan shared 300 mangoes in the ratio of 3 : 7. How many mangoes did each get?
8. Two villages shared 24 000 sacks of rice in the ratio of 7 : 9. Find the difference in sacks of rice between the two villages.

9. In a certain community, the ratio of engineers to doctors is 18 : 13. If there are 432 engineers, how many doctors are in that community?

10. In a farm of 900 hectares, the ratio of hectares of rice to that of maize is 4 : 5. How many hectares of the farm are occupied by each plant?

11. The ratio of the number of goats to the number of sheep is 9 : 16. Find the number of sheep if there are 450 goats.

12. A rope of 95 metres long was divided into two pieces in the ratio of 2 : 3. Find the length of each piece.

13. If L : S = 12 : 13, find the value of:
    (a) L given S = 312.
    (b) S given L = 312.

14. The ratio of the number of periods for Mathematics to Science is 5 : 3. Find the number of periods for Science if the total number of periods is 96.

15. The departments of Kiswahili and English shared 1 800 books in the ratio of 5 : 7. How many books did each department get?

16. The ratio of the number of girls to boys in Standard Six is 11 : 12. The class has a total of 115 pupils. How many pupils in the class are:
    (a) Girls?
    (b) Boys?

17. A certain primary school has a total of 48 teachers. If the ratio of male to female teachers is 3 : 5, find the number of female teachers.

18. The ratio of cows in two regions is 88 : 43. If the regions have a total of 97 988 cows, find the number of cows in:
(a) The first region.
(b) The second region.

19. Hamza and Janeth shared a sum of money in the ratio of 17 : 19. If Janeth received 9 500 shillings, how much money did Hamza receive?

20. Two football teams shared 32 balls in a ratio of 3 : 5. Calculate the difference in number of balls between the two teams.

Finding the ratio of three quantities

Ratio can be of three quantities. The amount of each quantity in the ratio is obtained using the total share of the quantities. The ratio of three quantities can be calculated using the following steps:

1. Write the ratio of the three quantities A, B and C as A : B : C.
2. Add the given quantities, that is, A + B + C.
3. Write the ratio of each quantity to the sum of all quantities in fraction, that is,

$$\frac{A}{A+B+C}, \frac{B}{A+B+C} \text{ and } \frac{C}{A+B+C}.$$

4. If you are given a total share, example, D, multiply each fraction by the total share D as follows.

$$\frac{A}{A+B+C} \times D, \frac{B}{A+B+C} \times D \text{ and } \frac{C}{A+B+C} \times D.$$

The answers in step 4 represent the value of each quantity in the ratio of quantity A, B and C to the given share D.

Example 1

Write the ratio of 160 to 240 to 360 using a colon and simplify.

Solution

Arrange the numbers in the question, and then change the ratio from words to a colon as follows:

160 to 240 to 360 = 160 : 240 : 360

Divide each number involved in the ratio by the G.C.F of the numbers. In this case, G.C.F of 160, 240, 360 is 40. Thus, the ratio is:

$\frac{160}{40} : \frac{240}{40} : \frac{360}{40}$. This is the same as 4 : 6 : 9.

Therefore, the ratio of 160 to 240 to 360 is 4 : 6 : 9.

Example 2

Mr Samwel gave money to his three children: Mage, Elia and Imani, in the ratio of 3 : 5 : 7. If the total amount of money given was 1 950 000 shillings, how much money did each child get?

Solution

The ratio of money of each child to the total number of children is as follows:

Mage : Eli : Imani = 3 : 5 : 7

Add: 3 + 5 + 7 = 15

Write each number involved as a fraction of the total of all numbers.

The total amount of money given to the children is 1 950 000 shillings.

Multiply each fraction of the ratio of each child by the total amount of money as follows:

Mage: $\frac{3}{15} \times$ sh 1 950 000 = sh 390 000.

Eli: $\frac{5}{15} \times$ sh 1 950 000 = sh 650 000.

Imani: $\frac{7}{15} \times$ sh 1 950 000 = sh 910 000.

Therefore, Mage got 390 000 shillings, Eli got 650 000 shillings, and Imani got 910 000 shillings.

Example 3

If A : B : C = 3 : 15 : 7, find the value of C if A = 12 and B = 60.

Solution

Arrange the numbers in the ratio as follows:
12 : 60 : C = 3 : 15 : 7
Add all numbers in the ratio in both sides:
Left side gives: 12 + 60 + C = 72 + C.
Right side gives: 3 + 15 + 7 = 25.

Formulate an equation as follows:

$$\frac{C}{72 + C} = \frac{7}{25}.$$

Cross-multiply to get a simpler equation:
7(72 + C) = 25C
504 + 7C = 25C.

Collect like terms:
18C = 504.

Divide both sides by 18:

$$\frac{18C}{18} = \frac{504}{18}$$

C = 28.

Therefore, C = 28.

Exercise 3

Answer the following questions:

1. Simplify and write the following ratios using colons:
   (a) 15 to 18 to 24
   (b) 120 to 150 to 240
   (c) 360 to 450 to 720
   (d) 40 to 50 to 80
   (e) 24 to 40 to 64
   (f) 288 to 576 to 1 080

2. Find the value of each unknown variable in the following questions:
   (a) 12 : 13 : 15 = Y : 416 : 480
   (b) 7 : 4 : 5 = H : G : K, if G = 216 and K = 270
   (c) 5 : 3 : 4 = 320 : M : 256
   (d) 3 : 5 : 7 = W : Z : P if Z = 180 and P = 252

3. Divide 1 870 kilograms in the ratio of 5 : 3 : 2.

4. The degree measures of a triangle are in the ratio of 5 : 7 : 8. Find the degree measure of each angle.

5. Divide 865 000 sacks in the ratio of 7 : 8 : 10.

6. Divide 64 000 litres of water in the ratio of 4 : 3 : 9.

7. Divide 960 pupils in groups in the ratio of 3 : 1 : 4.

8. Mercy, Ashura and Juma shared fruit seedlings in the ratio of 6 : 7 : 3. If Ashura got 84 seedlings, how many seedlings did Mercy and Juma get?

9. A teacher gave Judith, Sasha and Jerome 300 questions as home work. If they shared the questions in the ratio of 3 : 2 : 5, how many questions did each solve?

10. Mariam, Deus and Piala shared 280 exercise books in the following ratios: Deus received as twice as much as Piala, and Piala received twice as much as Mariam. How many exercise books did each receive?

11. Find the value of y if $2 : 3 : 5 = y + 1 : 60 : 100$.

12. A lorry carried tomatoes, carrots, and eggplants in the ratio of 8 : 7 : 3. If there were 350 carrots, find
(a) The number of tomatoes.
(b) The number of eggplants.

Simple ratios

A simple ratio shows the relative amount of two quantities. When the value of one quantity increases, the value of the other quantity increases too. When the value of one quantity decreases, the value of the other quantity decreases too.

Steps to find simple ratios:

1. Use the ratio of two quantities, that is, x and y which increase or decrease at the same time, as follows:
   x_1 represents the initial value of the quantity x.
   x_2 represents the value of the quantity x after an increase or decrease.
   y_1 represents the initial value of quantity y.
   y_2 represents the value of the quantity y after an increase or decrease.

2. Write and compare the ratio between those quantities, that is:

   $\frac{y_1}{y_2} = \frac{x_1}{x_2}$, $y_1 : y_2 = x_1 : x_2$

   If the values of x_1, y_1 and x_2 are known, you may obtain the value of y_2.
   Thus, $y_2 = \frac{y_1 x_2}{x_1}$. Also, if the values of x_1, y_1 and y_2 are known, you may obtain the value of x_2. Thus, $x_2 = \frac{y_2 x_1}{y_1}$.

Example 1

If the price of 8 kilograms of meat is sh 56 000, find the price of 6 kilograms of meat.

Solution

Find the price of 1 kilogram of meat as follows:

$$\text{The price of 1 kg} = \frac{\text{The price of 8 kg of meat}}{\text{number of kilograms}}$$

$$= \frac{\text{sh } 56\ 000}{8} = \text{sh } 7\ 000.$$

$$\text{The price of 6 kg} = 6\text{ kg} \times \frac{\text{sh } 7\ 000}{\text{kg}} = \text{sh } 42\ 000.$$

Alternative solution

Let x_1 and x_2 represent the kilograms of meat.
y_1 and y_2 represent the price of meat.

Use the expression $\frac{y_1}{y_2} = \frac{x_1}{x_2}$ to find the price of 6 kg of meat as follows:

$x_1 = 8$ kg, $x_2 = 6$ kg,

$y_1 = \text{sh } 56\ 000$, $y_2 = ?$

Find the value of y_2, which is the price of 6 kg of meat:

$$\frac{\text{sh } 56\ 000}{y_2} = \frac{8\text{ kg}}{6\text{ kg}}.$$

$$\text{Thus, } y_2 = \frac{6\text{ kg} \times \text{sh } 56\ 000}{8\text{ kg}} = \text{sh } 42\ 000.$$

Therefore, the price of 6 kilograms of meat is 42 000 shillings.

Example 2

The price of 5 books is sh 60 000. Find the price of 3 books of the same kind.

Solution

$$\text{The price of 1 book} = \frac{\text{The price of 5 books}}{\text{Total number of books}}$$

$$= \frac{\text{sh } 60\ 000}{5} = \text{sh } 12\ 000.$$

Thus, the price of 3 books = 3 × sh 12 000 = sh 36 000.

Alternative solution (Cross multiplication)

Let x_1 and x_2 represent the number of books.
y_1 and y_2 represent the price of books.

Use the expression $\frac{y_1}{y_2} = \frac{x_1}{x_2}$ to find the price of 3 books as follows:

$x_1 = 5,\ x_2 = 3,$
$y_1 = \text{sh } 60\,000,\ y_2 = ?$

Find the value of y_2, which is the price of 3 books:

$$\frac{\text{sh } 60\,000}{y_2} = \frac{5}{3}$$

$$y_2 = \frac{3 \times \text{sh } 60\,000}{5} = \text{sh } 36\,000.$$

Therefore, the price of three books of the same kind is sh 36 000.

Example 3

Twelve kilograms of sugar cost 30 000 shillings.
(a) Find the cost of 5 kilograms of sugar.
(b) How much money will be used to buy 65 kilograms of sugar?

Solution

(a) The cost of 1 kg $= \frac{\text{The cost of 12 kg of sugar}}{\text{Number of kilograms}}$

$$= \frac{\text{sh } 30\,000}{12} = \text{sh } 2\,500$$

Thus, the cost of 5 kg $= 5 \text{ kg} \times \frac{\text{sh } 2\,500}{1 \text{ kg}} = \text{sh } 12\,500.$

Therefore, the cost of 5 kilograms of sugar is 12 500 shillings.

(b) The amount of money $= 65 \text{ kg} \times \frac{\text{sh } 2\,500}{1 \text{ kg}} = \text{sh } 162\,500.$

Therefore, 162 500 shillings will be used to buy 65 kilograms of sugar.

Exercise 4

Answer the following questions:

1. Seven bottles of water have the capacity of 10.5 litres. Find the capacity of 24 bottles of the same type.

2. Hamis runs 36 kilometres in 2 hours. How many kilometres will he run in 4 hours?

3. If 3 hectres of a farm produce 75 sacks of maize, how many sacks will be produced in 18 hectres?

4. The cost of 15 exercise books is 7 500 shillings. Find the cost of 184 similar exercise books.

5. A vehicle travels a distance of 360 kilometres consuming 40 litres of petrol. In this journey, the vehicle consumes the same amount of petrol for each kilometre. How many kilometres will the vehicle travel with 63 litres?

6. The cost of 8 kilograms of beans is 19 200 shillings. Find the cost of 6 kilograms of the same beans.

7. Three hundred and seventy-five kilograms of peanuts are given to pupils for 3 days. How many kilograms will the pupils get for 48 days?

8. The cost of hiring a vehicle for a 70-kilometre journey is 140 000 shillings. Find the cost for hiring the vehicle for a 148-kilometre journery.

9. Eighteen sacks of wheat have the mass of 540 kilograms. What will be the mass of 56 sacks of the same kind of wheat?

10. Four pupils can clean 8 classrooms of the same size. How many classrooms of the same size can be cleaned by 70 pupils?

11. If 1 140 bricks are made using 38 bags of cement,
(a) how many bricks will be made with 95 bags of cement?
(b) how many bags of cement can make 13 740 bricks?

12. Six litres of milk are sold for 18 000 shillings. Find:
(a) The cost of 76 litres of milk.
(b) The number of litres of milk that will be sold for 468 000 shillings.

13. The cost of five school bags is 140 000 shillings. Calculate the cost of 17 bags of the same kind, if the bags are sold at the same price.

14. The price of 18 kilograms of rice is 32 400 shillings. Find the cost of 29 kilograms of rice of the same type.

15. A bundle of tomatoes costs 1 500 shillings. Find the cost of 16 bundles of tomatoes.

16. The cost of 32 exercise books is 80 000 shillings. Find the cost of 4 dozens of similar exercise books.

17. A total of 270 exercise books are shared equally between 30 pupils. How many exercise books of the same type will be shared by 16 pupils?

18. The cost of 15 kilograms of goat meat is 108 000 shillings. Find the cost of 32 kilograms of the same meat.

19. The cost of 402 mangoes is 221 100 shillings. If each mango is sold at the same cost, find the cost of 865 mangoes of the same kind.

20. A vehicle spends 3 hours to travel a distance of 240 kilometres. If the vehicle spends the same time for each kilometre, how long will the vehicle spend to travel 920 kilometres?

Mixed ratios

Example 1

A farm is cultivated in 4 days by 12 people with the same ability. In how many days will the same farm be cultivated by 16 people with the same ability?

Solution

Twelve people cultivate the farm in 4 days
1 person will cultivate the farm in $12 \times 4 = 48$ days.
16 people will cultivate the farm in $48 \div 16 = 3$ days.
Therefore, 16 people will cultivate the farm in 3 days .

Alternative solution (cross-multiplication method)

Arrange as follows:

People	Farm	Days
12	1	4
16	1	x

x represents the number of days that will be spent by 16 people to cultivate the farm. Use cross-multiplication:

People	Farm	Days
12	1	4
16	1	x

Multiply by following the arrows direction as shown below:

$12 \times 1 \times 4 = 16 \times 1 \times x$

$48 = 16x.$

Divide both sides by 16:

$$\frac{16x}{16} = \frac{48}{16}$$

$x = 3.$

Therefore, 16 people will cultivate the farm in 3 days.

Example 2

Six workers received 300 000 shillings in 4 working days. How much will 28 workers receive in 2 working days?

Solution

Arrange the ratios of three items as follows:

Workers	Income	Days
6	300 000	4
28	y	2

y represents the income of 28 workers for 2 working days
Multiply by following the arrowsdirection as follows:

$6 \times y \times 4 = 28 \times 300\ 000 \times 2$

$24y = 28 \times 300\ 000 \times 2.$

Divide each side by 24:

$$\frac{24y}{24} = \frac{28 \times 300\ 000 \times 2}{24}$$

$$y = \frac{28 \times 300\ 000 \times 2}{24} = \text{sh } 700\ 000.$$

Therefore, 28 workers will get 700 000 shillings in two working days.

Exercise 5

Answer the following questions:

1. In 8 days, a total of 90 pupils can eat 240 kilograms of rice. In how many days will 180 pupils eat the same number of kilograms of rice if each pupil eats the same amount?

2. A school has enough food for 150 pupils to eat in 10 days. In how many days will 300 pupils eat the same amount of food?

3. A total of 480 000 shillings was paid to 8 workers for working 2 days. How much money will 12 workers with the same ability earn if they perform the same work for 6 days?

4. A house wall is painted in 120 minutes by 2 people. How long will 3 people with the same ability take to paint the same wall?

5. A total of 20 boxes are filled with fruits. Each box has 18 fruits. How many boxes are needed if each box is to be filled with 12 fruits?

6. If I spend 1 800 shillings everyday, I have enough money for 40 days. For how many days will the money last if I spend 1 500 shillings everyday?

7. A total of 360 hectares are cultivated in 20 days by 6 villages. If there is an addition of 4 villages, for how many days will the same number of hectares be cultivated?

8. A family has enough food for 12 people with the same eating ability for 48 days. In how many days will 16 people eat the same amount of food?

9. A farm of 4 hectares is cultivated in 16 days by 24 people with the same ability. In how many days will 48 people with the same ability cultivate the same farm?

10. A total of 4 kilograms of rice is consumed by 16 people in 2 days. In how many days will 8 people with the same eating ability consume the same number of kilograms of rice?

11. A number of 5 people with the same ability can cultivate 1 hectare of a farm for 10 days. How many people will cultivate the same farm in 5 days?

12. In 4 days, 12 prisoners slash 3 600 square metres of a farm land. How many square metres will 20 prisoners with the same ability slash for 8 days?

13. A total of 8 hectares produce 200 sacks of maize. How many sacks of maize will be produced in 5 hectares with the same level of production?

14. Shamsa's age is four times the age of her child. If the sum of their ages is 120 years. Find:
(a) The age of Shamsa.
(b) The age of her child.

15. Kenedy's age is three times the age of Elias. Five years ago, the sum of their ages was 70 years. Find the current age of Kenedy.

16. Two taps of the same capacity fill a water barrel for 45 minutes. For how long will 5 similar taps fill the same barrel?

17. A carpenter makes 6 tables in 3 days. How many days will the carpenter spend making 36 tables?

18. A total amount of 7 200 000 shillings was given to 12 workers for 4 working days. How many shillings will 6 workers get for 3 working days if all workers are paid the same amount?

19. The school garden is cultivated for 20 days by 5 pupils with the same ability:
(a) How many days will 4 pupils spend cultivating the same garden?
(b) How many pupils can cultivate the same garden in 2 days?

20. A farm of 12 hectares can be cultivated in 36 days by 10 people. In how many days will 5 people of the same capacity spend cultivating a 15-hectare farm?

Revision exercise

Answer the following questions:
In question 1 up to 9, write the ratios using the colon, and then simplify them.

1. 90 to 120.

2. 75 to 225.

3. 50 to 375.

4. 80 to 35.

5. 70 to 100.

6. 180 to 210 to 360.

7. 375 to 475 to 625.

8. 60 to 90 to 360.

9. 32 to 80 to 98.

10. If R : S = 4 : 9, find the value of S, given R = 292.

11. If D : E : F = 8 : 9 : 11, find the value of E if D = 256 and F = 352.

12. Divide 360 000 metres in the ratio of 9 : 12 : 15.

13. If 3 : 4 : 5 = 36 : G : 60, find the value of G.

14. The ages of 3 teachers are 54, 32 and 40 years. Write the ratio of their ages starting from the oldest to youngest:
    (a) In words.
    (b) Using colons.

15. A farm of 945 hectares is divided into two parts in a ratio of 7 : 8. How many hectares does each part have?

16. A total of 18 bundles have 2 880 grams. How many grams do 36 bundles of similar mass have?

17. Petro, Anna and Abdallah shared 150 000 shillings in the ratio of 4 : 5 : 6. Who got the largest share, and by what amount does this largest share exceed the smallest share?

18. The school has 520 pupils. The ratio of girls to boys is 5 : 3. How many boys and girls does the school have?

19. The angles of a triangle are in the ratio of 9 : 10 : 11. Find the degree measure of each angle.

20. A board 3 metres long is sold for 37 500 shillings. How much does a 4-metre long board of the same type cost?

21. A total of 4 packages of second hand cotton clothes weigh 240 kilograms. What is the weight of 7 packages of similar second hand cotton clothes?

22. The price of 6 bundles of oranges is 99 000 shillings. Find the price of 29 bundles of similar oranges.

23. The price of 12 bunches of banana is 216 000 shillings. Find the price of 9 bunches of banana of the same type.

24. A total of 270 sacks of maize are produced from 18 hectares. How many sacks will be produced from 12 hectares with the same production capacity?

25. A farm of 6 hectares is cultivated by 5 people for 8 days. How long will it take for 4 people to cultivate 9 hectares

26. A total number of 180 pupils consume 360 kilograms of flour in 5 days. For how many days will 75 pupils with the same eating ability consume the same amount of flour?

27. A total amount of 4 800 000 shillings was given to 12 workers for 6 work days. How much money will 24 workers with the same working ability receive if they work for 8 days?

Find the value of each letter in the tables provided in questions 28 up to 30, if the rows have equivalent ratios:

28.

4	11
8	m
n	33
16	p
20	q
r	66

29.

s	7
10	t
u	21
20	28
25	v
w	42

30.

3	17
6	K
9	L
M	68
15	N
Q	102

Summary

1. Ratio is the relationship between two or more quantities or numbers.
2. A dozen is the collection of twelve similar items.
3. The numbers involved in ratios do not represent the actual value of something contained, instead they represent the number of times one quantity is contained with respect to the other quantity.
4. The larger the number in the ratio the higher the value of the quantity it represents in the ratio.

Chapter Seven

Exponents and square roots of numbers

Introduction

In Standard Five, you learnt to compute square numbers, square of numbers, and square root of square numbers. In this chapter, you will learn operations on exponents, simplification of various algebraic expressions involving exponents. You will also learn to compute the square of numbers not exceeding four digits and the square root of numbers not exceeding six digits. The competencies gained will help you to write large numbers in simple form and easily determine the number of living organisms such as bacteria and viruses which increase exponentially in a short period of time.

Exponents of numbers

An exponent is a quantity that shows how many times a number is repeated in multiplication. The number in exponential form is written as "a^n", where, "a" is the base and "n" is an exponent. Also "a^n" is called a power. The exponent of a number is written on the top right of the base. For example, $3 \times 3 \times 3 \times 3 \times 3$ in exponential form is written as 3^5 and it is read as three exponent five or three power five.

Writing numbers in exponential form

A number with the same factors can be written as a product of its factors. The product of the factors can be written in exponential form as shown in the following example:

Example

(a) $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6$

(b) $5 \times 5 \times 5 \times 5 = 5^4$

(c) $\left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) = \left(\frac{3}{5}\right)^4$

(d) $y \times y \times y \times y \times y = y^5$

Exercise 1

Answer the following questions:

1. Write the product of the following numbers in exponential form:

(a) $4 \times 4 \times 4 \times 4 \times 4 =$

(b) $\left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) =$

(c) $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 =$

(d) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 =$

(e) $8 \times 8 \times 8 \times 8 \times 8 =$

2. Write the following exponents in expanded form:

(a) 13^4 (c) $(-7)^4$ (e) $\left(-\frac{1}{9}\right)^3$

(b) $\left(\frac{11}{12}\right)^2$ (d) 17^3 (f) 3^3

3. Calculate the following exponents:

(a) 2^4 (b) $\left(\frac{3}{4}\right)^2$ (c) $(-5)^3$

4. Write the following numbers in exponential form:

(a) 512 (c) 250 (e) 729

(b) 342 (d) 3 125 (f) 225

Multiplication of exponential numbers (powers)

Consider two pairs of powers, that is, a^n, a^m and b^n, b^m. where n and m are integers.

(a) a^n and a^m are powers with the same base and different exponents.

Thus, $a^n \times a^m = a^{n+m}$.

(b) a^n and b^m are powers with different bases and exponents.
Thus, $a^n \times b^m \neq a^{n+m}$ or $a^n \times b^m \neq b^{m+n}$.

(c) a^n and b^n are powers with the same exponent but different bases.
Thus, $a^n \times b^n = (a \times b)^n$.

Example 1

Simplify the following expressions, and then write the answers in exponential form:

(a) $5^8 \times 5^3$

(b) $\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^5$

(c) $(-7)^4 \times (-7)^2$

(d) $2^3 \times 4^3$

Solution

(a) $5^8 \times 5^3 = 5^{8+3}$
$= 5^{11}$.

Therefore, $5^8 \times 5^3 = 5^{11}$.

(b) $\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^5 = \left(\frac{2}{5}\right)^{3+5}$
$= \left(\frac{2}{5}\right)^8$.

Therefore, $\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^5 = \left(\frac{2}{5}\right)^8$.

(c) $(-7)^4 \times (-7)^2 = (-7)^{4+2}$
$= (-7)^6$.

Therefore, $(-7)^4 \times (-7)^2 = (-7)^6$.

(d) $2^3 \times 4^3 = (2 \times 4)^3$
$= 8^3$.

Therefore, $2^3 \times 4^3 = 8^3$.

Carefully study the following equations:

(a) $(a^n)^2 = a^n \times a^n$
$= a^{n+n}$
$= a^{2n}$
$= a^{n \times 2}$.

(b) $(a^n)^3 = a^n \times a^n \times a^n$
$= a^{n+n+n}$
$= a^{3n}$
$= a^{n \times 3}$.

By considering the results obtained from parts (a) and (b) above, we conclude that $(a^n)^m = a^{n \times m}$.

Example 2

Simplify the following expressions, and then write answers in exponential form:

(a) $(3^5)^4$ (b) $(5^2)^3$

Solution

(a) $(3^5)^4 = 3^{5 \times 4}$

$= 3^{20}$.

Therefore, $(3^5)^4 = 3^{20}$.

(b) $(5^2)^3 = 5^{2 \times 3}$

$= 5^6$.

Therefore, $(5^2)^3 = 5^6$.

Exercise 2

Simplify the following expressions, and then write answers in exponential form:

1. $3^2 \times 3^4 \times 3^8$
2. $6^{15} \times 6^{10}$
3. $a^3 \times a^2$
4. $7^4 \times 7^5 \times 7^2$
5. $\left(\frac{4}{9}\right)^2 \times \left(\frac{4}{9}\right)^5 \times \left(\frac{4}{9}\right)^7$
6. $(11^4)^2$
7. $(-5)^3 \times (-5)^5 \times (-5)^2 \times (-5)$
8. $6^4 \times 6^2$
9. $\left(-\frac{5}{11}\right)^3 \times \left(-\frac{5}{11}\right)^3 \times \left(-\frac{5}{11}\right)^2$
10. $(7^3)^4$
11. $\left(-\frac{5}{13}\right)^2 \times \left(-\frac{5}{13}\right)^3 \times \left(-\frac{5}{13}\right)^6$
12. $m^4 \times m^3 \times m^2 \times m$
13. $2^6 \times 2^4 \times 2^5 \times 2^2$
14. $(5^3)^2 \times (5^4)^3$

15. $(p^3)^7 \times (p^2)^4$

16. $5^3 \times 8^3$

17. $\left(\left(\frac{12}{13}\right)^4\right)^5$

18. $\left(-\frac{13}{17}\right)^4 \times \left(-\frac{13}{17}\right)^8 \times \left(-\frac{13}{17}\right)^5$

19. $5^4 \times 5^5 \times 5^7 \times 5^3$

20. $4^8 \times 4^5 \times 4^9 \times 4^3 \times 4^2$

Division of exponential numbers

Consider exponential numbers with the same base, for example " a^n" and " a^m". You can simply divide the numbers with the same base as follows:

$$a^5 \div a^3 = \frac{a^5}{a^3}$$

$$= \frac{a \times a \times \cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a}}$$

$$= a^2.$$

Thus, $a^5 \div a^3 = a^{5-3}$

$= a^2.$

This example shows that, you have to subtract the exponents when dividing numbers with the same base.

Therefore, $a^n \div a^m = a^{n-m}$.

Exponential numbers with the same exponents and different bases such as "a^n" and "b^n" can be divided as follows:

$$a^n \div b^n = \frac{a^n}{b^n}$$

$$= \left(\frac{a}{b}\right)^n.$$

Example 1

Simplify the following expressions, and then write the answers in exponential form:

(a) $7^9 \div 7^5$ (b) $\left(\frac{2}{7}\right)^7 \div \left(\frac{2}{7}\right)^3$ (c) $5^4 \div 3^4$

Solution

(a) $7^9 \div 7^5 = 7^{9-5}$

$= 7^4$.

Therefore, $7^9 \div 7^5 = 7^4$.

(b) $\left(\frac{2}{7}\right)^7 \div \left(\frac{2}{7}\right)^3 = \left(\frac{2}{7}\right)^{7-3}$

$= \left(\frac{2}{7}\right)^4$.

Therefore, $\left(\frac{2}{7}\right)^7 \div \left(\frac{2}{7}\right)^3 = \left(\frac{2}{7}\right)^4$.

(c) $5^4 \div 3^4 = \frac{5^4}{3^4}$

$= \left(\frac{5}{3}\right)^4$.

Therefore, $5^4 \div 3^4 = \left(\frac{5}{3}\right)^4$.

Zero exponent

When dividing numbers with the same base and exponent, you will obtain a base with a zero exponent. The following example shows the value of a base with a zero exponent.

Example 2

$a^3 \div a^3 = \frac{a^3}{a^3}$

$= \frac{\cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a}}$

$= 1$.

Remember that $a^3 \div a^3 = a^{3-3}$

$= a^0$.

Therefore, $a^3 \div a^3 = 1$. This shows that, $a^0 = 1$.

Simplification of exponential expressions

Example 3

Simplify the following expressions, and then write the answers in exponential form:

(a) $\frac{2^3 \times 3^4 \times 4}{3 \times 32}$

(b) $\frac{8a^5b^3 + 4a^3b^4}{2a^3b^3}$

Solution

(a) Find the prime factors of 32 and 4; then write their products in exponential form:

$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$; $4 = 2 \times 2 = 2^2$.

$$\frac{2^3 \times 3^4 \times 4}{3 \times 32} = \frac{2^3 \times 3^4 \times 2^2}{3 \times 2^5}$$

$$= \frac{2^{3+2} \times 3^4}{3 \times 2^5}$$

$$= \frac{2^5 \times 3^4}{3 \times 2^5}$$

$$= \frac{2^5}{2^5} \times \frac{3^4}{3} = 2^{(5-5)} \times 3^{(4-1)}$$

$$= 2^0 \times 3^3 = 1 \times 3^3 = 3^3.$$

Therefore, $\frac{2^3 \times 3^4 \times 4}{3 \times 32} = 3^3$.

(b) This example shows how to simplify exponential algebraic expressions.

$$\frac{8a^5b^3 + 4a^3b^4}{2a^3b^3} = \frac{8a^5b^3}{2a^3b^3} + \frac{4a^3b^4}{2a^3b^3}$$

$$= \frac{8}{2}\left(\frac{a^5}{a^3} \times \frac{b^3}{b^3}\right) + \frac{4}{2}\left(\frac{a^3}{a^3} \times \frac{b^4}{b^3}\right)$$

$$= 4(a^{5-3} \times b^{3-3}) + 2(a^{3-3} \times b^{4-3})$$

$$= 4(a^2 \times b^0) + 2(a^0 \times b)$$

$$= 4(a^2 \times 1) + 2(1 \times b)$$

$$= 4a^2 + 2b$$

$$= 2(2a^2 + b).$$

Alternative solution 1

$$\frac{8a^5b^3 + 4a^3b^4}{2a^3b^3} = \frac{8a^5b^3}{2a^3b^3} + \frac{4a^3b^4}{2a^3b^3}$$

$$= 4a^2(1) + 2(1)b$$

$$= 4a^2 + 2b$$

$$= 2(2a^2 + b).$$

Therefore, $\frac{8a^5b^3 + 4a^3b^4}{2a^3b^3} = 2(2a^2 + b)$.

Alternative solution 2

$$\frac{8a^5b^3 + 4a^3b^4}{2a^3b^3} = \frac{4a^3b^3(2a^2 + b)}{2a^3b^3}$$
$$= 2(2a^2 + b).$$

Therefore, $\frac{8a^5b^3 + 4a^3b^4}{2a^3b^3} = 2(2a^2 + b)$.

Exercise 3

Answer the following questions:

1. Simplify the following expressions, and then write the answers in exponential form:

(a) $7^5 \div 7^2$
(b) $5^5 \div 5^3$
(c) $15^8 \div 15^5$
(d) $3^{13} \div 3^{11}$
(e) $4^{30} \div 4^{12}$
(f) $100^{18} \div 100^{12}$
(g) $(17^{12} \div 17^5) \times (17^2)^3$
(h) $(8^6 \div 8^2) \times (8^{12} \div 8^4)$
(i) $(9^{12} \div 9^4) \div 9^7$
(j) $6^4 \div 2^4$

2. Simplify the following algebraic expressions:

(a) $\frac{24p^6q^4}{8p^4q^3}$
(b) $\frac{2a^4b^4 + a^2b^2}{a^2b^2}$
(c) $\frac{4m^6n^4 - 2m^3n^3}{2m^3n^3}$
(d) $\frac{72y^8w^6z^{12}}{8y^4w^3z^6}$
(e) $\frac{64c^4d^2e^6 - 16c^2d^2e^2}{8c^2d^2e^2}$
(f) $\frac{4d^8b^8 + d^4b^4}{d^4b^4}$

3. Find the value of $\frac{x^2y^2}{xy}$ given that x = 2 and y = 4.

4. Find the value of $\frac{a^2b^2 + ab^2}{a^2b^2}$, if a = 1 and b = 2.

5. Find the value of $\frac{x^2y^4 + x^4y^2}{x^2y^2}$, if x = 1 and y = 3.

6. Find the value of $\frac{t^4u^3 + v^2w^4}{tuvw}$, if t = 2, u = 1, v = 3 and w = 2.

7. Find the value of $\frac{(a^2 + b^2)^2 + (a^2 - b^2)^2}{(a - b)^2}$, if a = 4 and b = 2.

8. Find the value of $\frac{(m^5)^3 n^2 + n^6 m}{n m^4 - m^7}$, if $n = 4$ and $m = 1$.

9. Find the value of $\frac{(r - s)^3 - (t^2 - s)^2}{t^5}$, if $r = 3$, $s = 1$ and $t = 2$.

10. Find the value of $\frac{(ab)^3 + (b^2 c^2)^4}{abc}$, if $a = 5$, $b = 2$ and $c = 3$.

Square root of a square number

The square root of a square number is the opposite of the second power of the number. Finding the square root of a square number is the same as finding the base of the second power number. The square root of a square number is represented by the symbol $\sqrt{\ }$. The symbol $\sqrt{\ }$ means that a number is raised to the half power, that is $\sqrt{a} = a^{½}$.

Finding the square root of a square number with not more than three digits

The computation of a square root of a square number is done by using different methods including tree diagrams, factorization and grouping of digits.

Example 1

Find the square root of 256 by using a tree diagram.

Solution

Use the following steps:

1. Draw a tree diagram and obtain all prime factors

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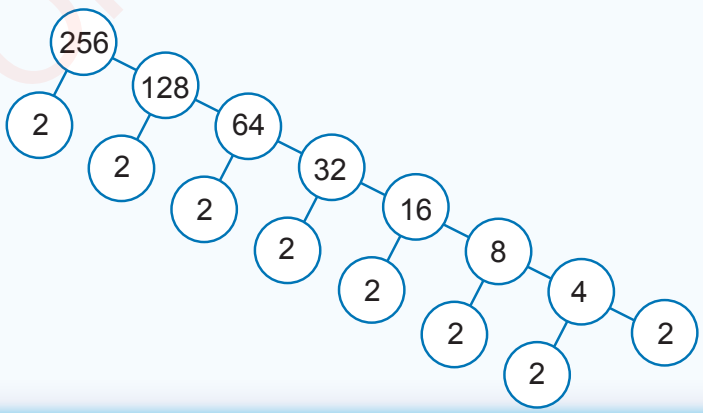


2. Write the square root of 256 as a product of prime factors:
$\sqrt{256} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$
3. Arrange in pairs the product of the same prime factors:
$\sqrt{256} = \sqrt{2 \times 2} \times \sqrt{2 \times 2} \times \sqrt{2 \times 2} \times \sqrt{2 \times 2}$
4. Take one factor from each pair of prime factors, and then multiply. The product of these factors will be the square root of 256. That is,
$\sqrt{256} = 2 \times 2 \times 2 \times 2$
$= 16.$
Therefore, the square root of 256 is 16.

Example 2

Find the square root of 196 using the method of prime factors.

Solution

Use the following steps:

1. Divide 196 as shown below in order to get all its prime factors:

2	196
2	98
7	49
7	7
	1

2. Write the square root of 196 as a product of its prime factors:
$\sqrt{196} = \sqrt{2 \times 2 \times 7 \times 7}$
3. Arrange in pairs the product of the same prime factors:
$\sqrt{196} = \sqrt{2 \times 2} \times \sqrt{7 \times 7}$
4. Take one factor from each pair of prime factors, and then multiply them. The product of these factors is the square root of 196.
Thus, $\sqrt{196} = 2 \times 7 = 14.$

Therefore, the square root of 196 is 14.

Example 3

Find the square root of 625 by grouping digits.

Solution

Use the following steps:

1. Group the digits into pairs from right.	$\sqrt{6\ 25}$
2. Find a number which when multiplied by itself the product is 6 or it approaches 6 and does not exceed 6. The required number is 2 since $2 \times 2 = 4$.	
3. Write 2 on top of 6 in the answer's position. Also, write 2 to the left as a divisor.	$\begin{array}{r} 2 \\ 2\overline{)\,6\ 25} \end{array}$
4. Multiply 2 of the part of the answer by 2 of the part of the divisor, $2 \times 2 = 4$, and then subtract their product from 6.	$\begin{array}{r} 2 \\ 2\overline{)\,6\ 25} \\ -4 \\ \hline 2 \end{array}$
5. Bring down the next two digits grouped from right and write them in front of 2 to get 225. Also, add 2 of the divisor and 2 of part of the answer to get a new divisor $2 + 2 = 4$. Write 4 on the left of 225.	$\begin{array}{r} 2 \\ 2\overline{)\,6\ 25} \\ -4 \\ \hline 4\ \ 2\ 25 \end{array}$
6. Find a number to be written to the right of 2 in the answer position. The same number should be written in front of a new divisor. This number is multiplied by a new divisor to get 225 or a product close to 225. The required number is 5. The new divisor becomes 45 and the value in the answer will be 25. Multiply 45 by 5 of part of the answer, $5 \times 45 = 225$. Write 225 below 225, and then subtract to get 0.	$\begin{array}{r} 2\ 5 \\ 2\overline{)\,6\ 25} \\ -4 \\ \hline 45\ \ 2\ 25 \\ -2\ 25 \\ \hline \quad\ \ 0 \end{array}$
Therefore, the square root of 625 is 25.	

Exercise 4

Answer the following questions:

1. Compute the following square roots by grouping the digits:
   (a) $\sqrt{196}$
   (b) $\sqrt{289}$
   (c) $\sqrt{400}$
   (d) $\sqrt{900}$
   (e) $\sqrt{576}$
   (f) $\sqrt{529}$
   (g) $\sqrt{484}$
   (h) $\sqrt{441}$

2. Compute the following square roots using tree diagram:
   (a) $\sqrt{361}$
   (b) $\sqrt{784}$
   (c) $\sqrt{144}$
   (d) $\sqrt{841}$
   (e) $\sqrt{324}$
   (f) $\sqrt{100}$
   (g) $\sqrt{169}$
   (h) $\sqrt{625}$

3. Compute the following square roots by using prime factors:
   (a) $\sqrt{676}$
   (b) $\sqrt{729}$
   (c) $\sqrt{121}$
   (d) $\sqrt{841}$

Finding the square root of a square number with not more than six digits

Example 1

Find the square root of 1 296 by using prime factors.

Solution

Use the following steps:

1. Divide in order to get the prime factors of 1 296.

2	1 296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

2. Write the square root of 1 296 as a product of its prime factors:

$\sqrt{1\ 296} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$.

3. Arrange in pairs the product of the same prime factors:

$\sqrt{1\ 296} = \sqrt{2 \times 2} \times \sqrt{2 \times 2} \times \sqrt{3 \times 3} \times \sqrt{3 \times 3}$.

4. Take one factor from each pair of prime factors, then multiply. The product of these factors is the square root of 1 296.

$\sqrt{1\ 296} = 2 \times 2 \times 3 \times 3 = 36$.

Therefore, the square root of 1 296 is 36.

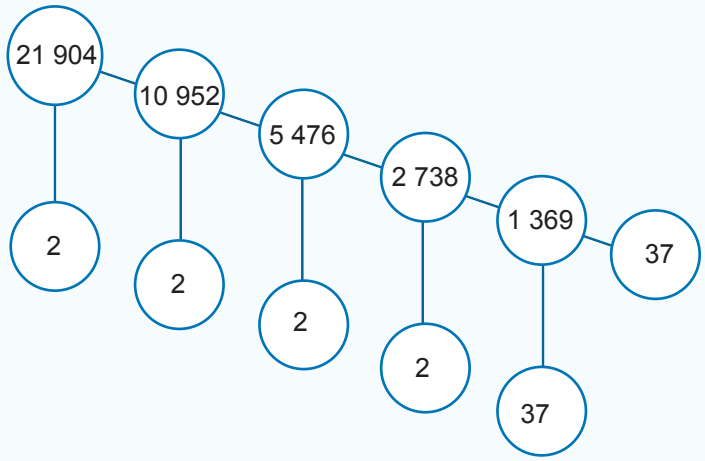
Example 2

Find the square root of 21 904 by using a tree diagram.

Solution

Use the following steps:

1. Draw a tree diagram and obtain all prime factors.



2. Write the square root of 21 904 as a product of its prime factors:
   $\sqrt{21\ 904} = \sqrt{2 \times 2 \times 2 \times 2 \times 37 \times 37}$.
3. Arrange in pairs the product of the same prime factors:
   $\sqrt{21\ 904} = \sqrt{2 \times 2} \times \sqrt{2 \times 2} \times \sqrt{37 \times 37}$.
4. Take one factor from each pair of prime factors, then multiply. The product of these factors is the square root of 21 904.
   $\sqrt{21\ 904} = 2 \times 2 \times 37 = 148$.

   Therefore, the square root of 21 904 is 148.

Example 3

Find the square root of 948 676 by grouping the digits.

Solution

Use the following steps:

1. Group the digits into pairs from the right.	$\sqrt{94\ 86\ 76}$
2. Find a number which when multiplied by itself the product is 94 or close to 94 and does not exceed 94. The required number is 9 since $9 \times 9 = 81$.	

3. Write 9 on top of 94 in the answer's position. Also, write 9 to the left as a divisor.	$\begin{array}{r} 9 \\ 9\overline{)\,94\,86\,76} \end{array}$
4. Multiply 9 of part of the answer by 9 of part of a divisor: $9 \times 9 = 81$, then subtract their product from 94 to get 13.	$\begin{array}{r} 9 \\ 9\overline{)\,94\,86\,76} \\ -81 \\ \hline 13 \end{array}$
5. Bring down the next two digits and write them to the right of 13 to get 1 386. Add 9 of the divisor and 9 of the answer to get a new divisor, 9 + 9 = 18. Write 18 on the left of 1 386	$\begin{array}{r} 9 \\ 9\overline{)\,94\,86\,76} \\ -81 \\ \hline 18 \quad 13\,86 \end{array}$
6. Find a number to be written to the right of 9 in the answer position. The same number should be written in front of a new divisor. This number is multiplied by a new divisor to get 1 386 or a product close to 1 386. The required number is 7. The new divisor becomes 187 and the value in the answer will be 97. Multiply 187 by 7 of part of the answer to get 1 309. Write 1 309 below 1 386 and subtract to get 77.	$\begin{array}{r} 9\ \ 7 \\ 9\overline{)\,94\,86\,76} \\ -81 \\ \hline 187 \quad 13\,86 \\ -13\,09 \\ \hline 77 \end{array}$
7. Bring down the last two digits and write them in front of 77 to get 7 776. Add 187 of the divisor and 7 of part of the answer to get a new divisor, which is 194. Write 194 on the left of 7 776.	$\begin{array}{r} 9\ \ 7\ \ 4 \\ 9\overline{)\,94\,86\,76} \\ -81 \\ \hline 187 \quad 13\,86 \\ -13\,09 \\ \hline 194 \quad 77\,76 \end{array}$
8. Find a number to be written to the right of 97 in the answer position. The same number should be written in front of a new divisor. This number is multiplied by a new divisor to get 7776 or a product close to 7 776. The required number is 4. The new divisor becomes 1944 and the value on the answer will be 974. Multiply 1944 by 4 of part of the answer to get 7 776. Write 7 776 below 7 776 and subtract to get 0	$\begin{array}{r} 9\ \ 7\ \ 4 \\ 9\overline{)\,94\,86\,76} \\ -81 \\ \hline 187 \quad 13\,86 \\ -13\,09 \\ \hline 1944 \quad 77\,76 \\ -77\,76 \\ \hline -\ \ - \end{array}$
Therefore, the square root of 948 676 is 974.	

Exercise 5

Answer the following questions:

1. Compute the following square roots by grouping the digits:
   (a) $\sqrt{9\ 801}$ (c) $\sqrt{38\ 416}$ (e) $\sqrt{69\ 696}$ (g) $\sqrt{964\ 324}$
   (b) $\sqrt{1\ 225}$ (d) $\sqrt{54\ 756}$ (f) $\sqrt{427\ 716}$ (h) $\sqrt{104\ 976}$

2. Compute the following square roots by using prime factors:
   (a) $\sqrt{9\ 216}$ (c) $\sqrt{96\ 100}$ (e) $\sqrt{859\ 329}$
   (b) $\sqrt{2\ 704}$ (d) $\sqrt{80\ 656}$ (f) $\sqrt{802\ 816}$

3. Compute the following square roots using tree diagrams:
   (a) $\sqrt{1\ 369}$ (c) $\sqrt{81\ 225}$ (e) $\sqrt{746\ 496}$
   (b) $\sqrt{7\ 744}$ (d) $\sqrt{38\ 416}$ (f) $\sqrt{459\ 684}$

Revision exercise

Answer the following questions:

1. Find the values of the following exponents:
   (a) 2^7 (c) 11^2
   (b) 9^3 (d) 5^4

2. Write the product of the following expressions in exponential form:
   (a) $6 \times 6 \times 6 \times 6$ (d) $5 \times 5 \times 7 \times 7 \times 7$
   (b) $t \times t$ (e) $a \times a \times a \times c \times c \times c \times c \times d$
   (c) $b \times b \times b \times b$

3. Write the following numbers in exponential form with prime bases:
   (a) 648 (b) 405 (c) 540 (d) 3 600

4. Simplify the following expressions. Write your answer in exponential form:
   (a) $4^2 \times 4^3 \times 4^8$ (d) $13^9 \times 13^{11}$ (g) $\left(\frac{3}{5}\right)^8 \div \left(\frac{3}{5}\right)^4$
   (b) $15^8 \div 15^8$ (e) $3^6 \times 3^5$ (h) $\frac{23^{12} \div 23^4}{23^4 \times 23^2}$
   (c) $7^4 \times 7^8 \times 7^5$ (f) $\frac{4^9 \times 4^8}{4^7 \times 4^2}$

5. Simplify the following expressions:

(a) $\frac{x^2y^3z^4}{xyz^2}$ (b) $\frac{30x^4y^6z^2}{15x^2y^5z}$ (c) $\frac{8a^6b^2 - 4a^4}{2a^4}$

6. Use the given values of the variables to find the values of the following expressions:

(a) $\frac{p^4q^3}{p^3q}$, if $p = 2$, $q = 4$.

(b) $\frac{2a^3b^2 - a^2b^2}{a^2b^2}$, if $a = 4$, $b = 2$.

(c) $\frac{5x^4y^2}{x^3y}$, if $x = 2$, $y = 3$.

7. Compute the following square roots by grouping the digits:

(a) $\sqrt{529}$ (d) $\sqrt{998\ 001}$

(b) $\sqrt{6\ 400}$ (e) $\sqrt{3\ 600}$

(c) $\sqrt{79\ 524}$ (f) $\sqrt{467\ 856}$

8. Compute the following square roots by using prime factors:

(a) $\sqrt{28\ 900}$

(b) $\sqrt{15\ 129}$

(c) $\sqrt{103\ 041}$

9. Compute the following square roots by using tree diagrams:

(a) $\sqrt{1\ 936}$

(b) $\sqrt{6\ 724}$

(c) $\sqrt{55\ 225}$

(d) $\sqrt{272\ 484}$

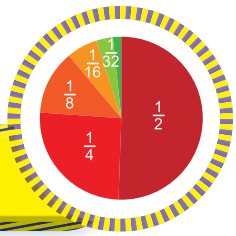
10. Find the value of $\sqrt{7\ 921}$.

Summary

1. An exponent is a number that shows how many times the base is repeated in multiplication. This number is written on the top right of the base.
2. A base is a number which is multiplied by itself several times.
3. A number in exponential form (a^n) has two parts, namely, base 'a' and exponent 'n'.
4. The square of a number is a number which shows that a base is multiplied by itself twice.
5. When dividing the same numbers with the same exponents, you will get a base with zero exponent and its value is 1.
6. A square root of a number is a number when multiplied by itself gives a square number.
7. A square root of a number is a factor of a square number.
8. The symbol "$\sqrt{\ }$" represents the words "square root".

Chapter Eight

Fractions and decimals



Introduction

In Standard Six, you learnt addition, subtraction, and multiplication of fractions and decimals. Also, you learnt simplification of fractions, changing fractions into percentages, percentages into decimals, and decimals into percentages. In this chapter, you will learn how to divide whole numbers by fractions, fractions by fractions, and fractions by decimals. The competencies gained in this chapter will help you to apply the concept of ratios, division of objects or make comparison of various objects in your daily life.

Exercise 1: Revision

Answer the following questions:

1. $\frac{7}{9} + \frac{2}{9} =$
2. $\frac{3}{10} + \frac{1}{4} =$
3. $2\frac{1}{4} + 3\frac{1}{5} =$
4. $21\frac{3}{5} + 8\frac{4}{7} =$
5. $\frac{1}{3} + \frac{14}{4} =$
6. $\frac{5}{8} - \frac{2}{5} =$
7. $9\frac{5}{12} - \frac{3}{7} =$
8. $19\frac{1}{4} - 6\frac{7}{8} =$
9. $12\frac{3}{5} - 8\frac{4}{9} =$
10. Subtract $10\frac{5}{8}$ from $15\frac{5}{12}$
11. $0.63 + 2.79 =$
12. $4.935 + 6.43 =$
13. $13.61 + 59.723 + 33.544 =$
14. $35.5 + 21.07 =$

15. $12.4 - 9.5 =$

16. $5.04 - 0.97 =$

17. $23.62 - 7.35 =$

18. $348.84 - 17.69 =$

19. $140.2 - 11.88 =$

20. $42.03 - 21.135 =$

21. $\frac{2}{5} \times \frac{5}{8} =$

22. $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{6} =$

23. $4\frac{2}{5} \times 2\frac{1}{2} \times \frac{2}{14} =$

24. $7\frac{1}{4} \times 2\frac{1}{4} =$

25. $19\frac{1}{5} \times 4\frac{1}{16} =$

26. $27.3 \times 4.1 =$

27. $10.6 \times 9.5 =$

28. $0.99 \times 0.25 =$

29. $0.065 \times 0.96 =$

30. $2.3 \times 4.8 \times 1.5 =$

31. Find the value of $\frac{1}{3} + \frac{1}{4} + \frac{1}{6}$.

32. Add 1.5 kilograms, 2.25 kilograms, 2.250 kilograms, and 0.15 kilograms.

33. Find the difference between $17\frac{1}{2}$ metres and $9\frac{1}{3}$ metres of a thread.

34. Halifa spent $\frac{3}{5}$ of his salary on buying food, $\frac{1}{6}$ on clothes and $\frac{1}{10}$ on cultivating vegetables. What fraction of his salary did he remain with?

35. A family uses $1\frac{3}{4}$ litres of milk everyday. How many litres of milk will the family use in 245 days?

36. A teacher's book has a length of 20.6 centimetres and a width of 12.5 centimetres. Find the surface area of the book.

37. Mzee Fimbo's farm has an area of 10 962.87 square metres. Mzee Maiga's farm has an area of 7 498.98 square metres. What is the difference in area of Mzee Maiga's farm and mzee Fimbo's farm?

38. Five pieces of wood have the following lengths: 0.85 metres, 0.75 metres, 0.9 metres, 0.99 metres and 0.97 metres. What is the total length of the pieces?

39. Find a number which when added to 49.75, the answer is 112.5.

40. Fatuma walked a distance of 69 kilometres in four days. She walked 20.78 kilometres on the first day, 16.7 kilometres on the second day, and 14.02 kilometres on the third day. Find the distance she walked on the fourth day?

Division of whole numbers by fractions

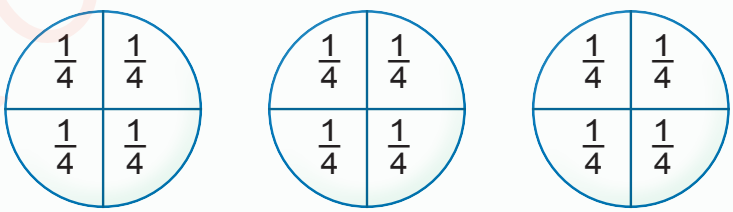
Activity: Techniques of dividing whole numbers by fractions

By using the given example, follow the procedures for dividing whole numbers by fractions.

$3 \div \frac{1}{4} =$

Steps

1. Consider the following three circles with equal sizes.



2. Divide each circle into four equal parts as shown. Each part makes a quarter of the circle. Count the number of quarters in all circles.

3. There are a total of 12 parts of the circles.
You can see that, $3 \div \frac{1}{4}$ means dividing 3 by $\frac{1}{4}$ which is the same as multiplying 3 by 4, which gives a total of 12 parts.
Therefore, $3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 12$.

From this activity, note the following:

1. The division sign changes to multiplication sign.
2. A denominator of the divisor changes to a numerator and the numerator changes to the denominator. That is, $\frac{1}{4}$ becomes $\frac{4}{1}$.

Example 1

$10 \div \frac{1}{2} =$

Solution

Write 10 as a fraction (with numerator and denominator): $10 \div \frac{1}{2} = \frac{10}{1} \div \frac{1}{2}$

Change the division sign (÷) into a multiplication (×) sign. Also, change the divisor, which is $\frac{1}{2}$ to its reciprocal, that is, $\frac{1}{2}$ will be $\frac{2}{1}$.

Thus, $\frac{10}{1} \div \frac{1}{2} = \frac{10}{1} \times \frac{2}{1}$

$= \frac{20}{1}$

$= 20.$

Therefore, $10 \div \frac{1}{2} = 20$.

Example 2

$200 \div 6\frac{1}{4} =$

Solution

Write 200 as a fraction, and then express the mixed number $6\frac{1}{4}$ into an improper fraction.

$200 \div \frac{25}{4} = \frac{200}{1} \div \frac{25}{4}$

Change the division sign into a multiplication sign, and then reciprocate the divisor to be $\frac{4}{25}$.

Hence, $\frac{200}{1} \div \frac{25}{4} = \frac{\overset{8}{\cancel{200}}}{1} \times \frac{4}{\cancel{25}\,1}$

$= 8 \times 4$

$= 32.$

Therefore, $200 \div 6\frac{1}{4} = 32.$

Example 3

$56 \div 2\frac{1}{8} =$

Solution

Write 56 as a fraction, and then change the mixed number $2\frac{1}{8}$ into an improper fraction.

$56 \div 2\frac{1}{8} = \frac{56}{1} \div \frac{17}{8}$

Change the division sign into a multiplication sign, and then change the divisor to be $\frac{8}{17}$:

Hence, $\frac{56}{1} \div \frac{17}{8} = \frac{56}{1} \times \frac{8}{17}$

$= \frac{448}{17}$

$= 26\frac{6}{17}.$

Therefore, $56 \div 2\frac{1}{8} = 26\frac{6}{17}.$

Exercise 2

Answer the following questions:

1. $6 \div \frac{2}{3} =$
2. $12 \div \frac{4}{7} =$
3. $9 \div \frac{5}{8} =$
4. $45 \div \frac{5}{12} =$
5. $22 \div \frac{11}{32} =$
6. $324 \div \frac{4}{11} =$
7. $121 \div \frac{11}{3} =$
8. $2\,010 \div \frac{30}{35} =$
9. $96 \div \frac{48}{13} =$
10. $132 \div \frac{12}{11} =$
11. $17 \div \frac{54}{9} =$
12. $168 \div \frac{16}{15} =$
13. $1 \div 6\frac{2}{3} =$
14. $9 \div 2\frac{7}{15} =$
15. $48 \div 3\frac{3}{4} =$
16. $255 \div 15\frac{5}{7} =$
17. $12 \div 37\frac{1}{2} =$
18. $10\,000 \div 10\frac{2}{5} =$
19. $880 \div 1\frac{1}{43} =$
20. $625 \div 12\frac{1}{2} =$

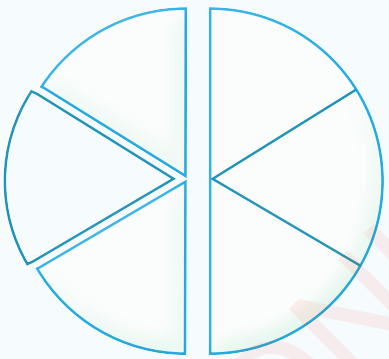
Division of fractions by whole numbers

Example 1

A teacher divided a half of a circular piece of paper into three equal parts. Three pupils were given one part each. What fraction of the circular piece of paper did each pupil get?

Solution

1. The instructions of the question can be written using numbers and division sign as follows:
   $\frac{1}{2} \div 3 =$



2. When each half of a circular piece of paper is divided into three equal parts, a total of six parts are obtained.

3. In fraction one part from the 6 parts of the circular piece of paper is written as $\frac{1}{6}$.

   Therefore, $\frac{1}{2} \div 3 = \frac{1}{6}$.

From this example, you can obtain $\frac{1}{6}$ from $\frac{1}{2} \div 3$, as follows:
The division sign should be changed to a multiplication sign, and 3 is written as its reciprocal and becomes $\frac{1}{3}$.

Thus, $\frac{1}{2} \div 3 = \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{6}$.

Therefore, each pupil got $\frac{1}{6}$ of the circular piece of paper.

Example 2

$\frac{2}{5} \div 2 =$

Solution

Write a whole number as a fraction with the denominator 1:

$\frac{2}{5} \div 2 = \frac{2}{5} \div \frac{2}{1}$

Multiply the dividend by the reciprocal of the divisor:

$$\frac{2}{5} \div 2 = \frac{2}{5} \times \frac{1}{2}$$
$$= \frac{2}{10}$$
$$= \frac{1}{5}.$$

Therefore, $\frac{2}{5} \div 2 = \frac{1}{5}$.

Example 3

$\frac{5}{3} \div 15 =$

Solution

Use the same steps as in Example 2:

$$\frac{5}{3} \div 15 = \frac{5}{3} \div \frac{15}{1}$$
$$= \frac{5}{3} \times \frac{1}{15}$$
$$= \frac{5}{45}$$
$$= \frac{1}{9}.$$

Therefore, $\frac{5}{3} \div 15 = \frac{1}{9}$.

Exercise 3

Answer the following questions:

1. $\frac{4}{5} \div 10 =$
2. $\frac{8}{11} \div 16 =$
3. $\frac{12}{13} \div 8 =$
4. $1\frac{5}{7} \div 6 =$
5. $\frac{121}{90} \div 55 =$
6. $8\frac{1}{10} \div 9 =$
7. $\frac{125}{3} \div 25 =$
8. $\frac{3}{20} \div 28 =$

9. $\frac{9}{17} \div 108 =$

10. $\frac{396}{5} \div 22 =$

11. $\frac{1}{64} \div 24 =$

12. $\frac{6}{20} \div 12 =$

13. $2\frac{1}{5} \div 33 =$

14. $\frac{5}{2} \div 46 =$

15. $7\frac{13}{31} \div 76 =$

16. $\frac{15}{7} \div 8 =$

17. $6\frac{2}{5} \div 9 =$

18. $\frac{53}{8} \div 7 =$

19. $\frac{6}{13} \div 15 =$

20. $\frac{15}{16} \div 38 =$

Division of fractions by fractions

By using diagrams, you can understand the concept of dividing fractions by fractions.

Example 1

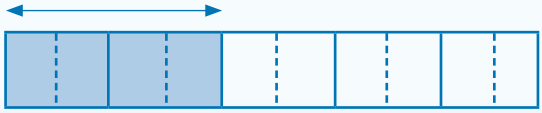
$\frac{2}{5} \div \frac{1}{10} =$

Solution

1. Show $\frac{2}{5}$ on the figure by drawing five equal parts, and then shade the two parts as shown in the following diagram:

2. $\frac{2}{5} \div \frac{1}{10}$ is the same as to say how many $\frac{1}{10}$ are there in $\frac{2}{5}$. Divide all parts of the diagram to obtain 10 equal parts as shown in the following diagram:

$\frac{2}{5}$



3. In step 2, the diagram shows that there are four $\frac{2}{5}$ in $\frac{1}{10}$.

By comparing the equations $\frac{2}{5} \div \frac{1}{10} = 4$ and $\frac{2}{5} \times \frac{10}{1} = 4$, you will note that the dividend does not change, but the division sign changes to multiplication sign and the denominator of the divisor becomes the numerator while its numerator becomes the denominator.

Therefore, $\frac{2}{5} \div \frac{1}{10} = 4$.

Based on the above results, the following formula is obtained:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}.$$

Example 2

$\frac{1}{3} \div \frac{1}{9} =$

Solution

Using the formula $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$,

$$\frac{1}{3} \div \frac{1}{9} = \frac{1}{3} \times \frac{9}{1}$$

$$= \frac{9}{3}$$

$$= 3.$$

Therefore, $\frac{1}{3} \div \frac{1}{9} = 3$.

Example 3

$\frac{65}{100} \div \frac{1}{25} =$

Solution

Using the formula $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$,

$$\frac{65}{100} \div \frac{1}{25} = \frac{65}{100} \times \frac{25}{1}$$

$$= \frac{65}{4}$$

$$= 16\frac{1}{4}.$$

Therefore, $\frac{65}{100} \div \frac{1}{25} = 16\frac{1}{4}$.

Example 4

$\frac{125}{1000} \div \frac{6}{50} =$

Solution

Using the formula $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$,

$$\frac{125}{1000} \div \frac{6}{50} = \frac{125}{1000} \times \frac{50}{6}$$

$$= \frac{25}{4} \times \frac{1}{6}$$

$$= \frac{25}{24}$$

$$= 1\frac{1}{24}.$$

Therefore, $\frac{125}{1000} \div \frac{6}{50} = 1\frac{1}{24}$.

Example 5

$\frac{1500}{605\ 500} \div \frac{5}{27} =$

Solution

Using the formula $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$,

$$\frac{1500}{605\ 500} \div \frac{5}{27} = \frac{1500}{605\ 500} \times \frac{27}{5}$$

$$= \frac{81}{6\ 055}.$$

Therefore, $\frac{1500}{605\ 500} \div \frac{5}{27} = \frac{81}{6\ 055}$.

Exercise 4

Answer the following questions:

1. $\frac{5}{12} \div \frac{3}{4} =$
2. $\frac{12}{48} \div \frac{3}{24} =$
3. $\frac{4}{13} \div \frac{8}{39} =$
4. $\frac{624}{1\,000} \div \frac{6}{8} =$
5. $5\frac{2}{3} \div \frac{21}{34} =$
6. $\frac{6\,611}{336\,611} \div \frac{44}{121} =$
7. $\frac{600}{122\,436} \div \frac{3}{4} =$
8. $\frac{132}{156} \div 6\frac{1}{4} =$
9. $\frac{7\,035}{800\,015} \div \frac{105}{1\,025} =$
10. $\frac{2}{9} \div \frac{25}{51} =$
11. $\frac{104}{112} \div \frac{13}{42} =$
12. $\frac{32}{117} \div \frac{4}{39} =$
13. $\frac{105}{1\,200} \div \frac{144}{196} =$
14. $\frac{1}{256} \div \frac{9}{80} =$
15. $\frac{1}{60} \div \frac{25}{27} =$
16. $10\frac{5}{12} \div 3\frac{3}{4} =$
17. $\frac{65}{3\,480} \div \frac{15}{7} =$
18. $\frac{1\,512}{245} \div \frac{18}{35} =$
19. $\frac{120}{1\,024} \div \frac{5}{8} =$
20. Divide $\frac{15}{20}$ by $\frac{4}{10}$.

Division of fractions by decimals

Decimals are fractions of 10 written by using a point (.) which separates a whole number and a fraction of 10 and its multiples. For example, $\frac{1}{10}$ is written in decimals as 0.1, $\frac{1}{100}$ as 0.01, $\frac{1}{1000}$ as 0.001, $\frac{3}{10}$ as 0.3, and $\frac{79}{100}$ as 0.79.

Also, mixed numbers can be written as decimals. For example, $5\frac{5}{10}$ can be written in decimals as 5.5, $1\frac{34}{100}$ as 1.34, and $61\frac{782}{1000}$ as 61.782.

When dividing fractions by decimals, first, convert the decimals into fractions, and then follow the procedures of dividing fractions by fractions using the formula $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.

Example 1

$\frac{1}{10} \div 0.3 =$

Solution

Convert 0.3 into fraction:

$$0.3 = \frac{3}{10}.$$

Thus, $\frac{1}{10} \div 0.3 = \frac{1}{10} \div \frac{3}{10}$.

Using the formula $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$:

$$\frac{1}{10} \div 0.3 = \frac{1}{10} \times \frac{10}{3}$$

$$= \frac{1}{3}.$$

Therefore, $\frac{1}{10} \div 0.3 = \frac{1}{3}$.

Example 2

$\frac{5}{8} \div 0.125 =$

Solution

Change 0.125 into fraction:

$$0.125 = \frac{125}{1000}.$$

Thus, $\frac{5}{8} \div 0.125 = \frac{5}{8} \div \frac{125}{1000}$.

Using the formula $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$:

$$\frac{5}{8} \div \frac{125}{1000} = \frac{5}{8} \times \frac{1000}{125}$$
$$= 5.$$

Therefore, $\frac{5}{8} \div 0.125 = 5$.

Example 3

$\frac{750}{15000} \div 10.15 =$

Solution

Change 10.15 into fraction:

$10.15 = 10\frac{15}{100}$

$= \frac{1015}{100}$.

Thus, $\frac{750}{15000} \div 10.15 = \frac{750}{15000} \div \frac{1015}{100}$

$$\frac{750}{15000} \div \frac{1015}{100} = \frac{750}{15000} \times \frac{100}{1015}$$
$$= \frac{5}{1015}$$
$$= \frac{1}{203}.$$

Therefore, $\frac{750}{15000} \div 10.15 = \frac{1}{203}$.

Exercise 5

Answer the following questions:

1. $\frac{300}{4} \div 0.02 =$
2. $\frac{75}{10} \div 0.6 =$
3. $\frac{14}{15} \div 0.05 =$
4. $\frac{3188}{65} \div 4.8 =$
5. $\frac{325}{2} \div 2.5 =$
6. $\frac{17}{3400} \div 0.051 =$
7. $2\frac{1}{2} \div 0.025 =$
8. $13\frac{7}{24} \div 2.75 =$

9. $12\frac{24}{100} \div 3.6 =$

10. $\frac{575}{625} \div 0.25 =$

11. $\frac{3}{700} \div 0.48 =$

12. $\frac{3}{8} \div 12.32645 =$

13. $\frac{120}{3120} \div 0.00001 =$

14. $7\frac{7}{56} \div 0.25 =$

15. $\frac{16}{175} \div 3.75 =$

16. $\frac{8}{2045} \div 1.25 =$

17. $\frac{95}{1256} \div 12.5 =$

18. $\frac{198}{12800} \div 16.8 =$

19. $15\frac{75}{112} \div 4.25 =$

20. $5\frac{35}{56} \div 0.5 =$

Word problems on division of fractions and decimals

Example 1

Sima bought 24 litres of milk and filled it in bottles of $\frac{3}{4}$ litres each. How many bottles did he use?

Solution

The milk bought was 24 litres.

Each bottle was filled with $\frac{3}{4}$ litres of milk.

Divide 24 litres by $\frac{3}{4}$ to obtain the number of bottles filled with milk:

$$24 \div \frac{3}{4} = \frac{24}{1} \times \frac{4}{3}$$

$$= 32.$$

Therefore, Sima used 32 bottles.

Example 2

A father ate $\frac{1}{6}$ of a loaf of bread and divided equally the remaining fraction to his 5 children. What fraction of the bread did each child get?

Steps

The remaining fraction of the bread was $1 - \frac{1}{6} = \frac{5}{6}$.
Number of children is 5.
Divide $\frac{5}{6}$ by 5 to get the fraction of bread each child got:

$$\frac{5}{6} \div 5 = \frac{5}{6} \times \frac{1}{5}$$
$$= \frac{5}{30}$$
$$= \frac{1}{6}.$$

Therefore, each child got $\frac{1}{6}$ of the bread.

Exercise 6

Answer the following questions:

1. Every pupil from Katale Primary School was asked to bring $2\frac{1}{2}$ kilograms of maize grains. If 85 kilograms of maize grains were collected, how many pupils brought the maize grains?

2. In a certain boarding school, each pupil usually eats $\frac{1}{8}$ kilograms of meat. How many pupils will eat 60 kilograms of meat?

3. The school area is $8\frac{1}{4}$ hectares. How many plots of size $2\frac{3}{4}$ hectares each can be obtained from this area?

4. A letter weighs 0.75 grams. How many letters of the same kind will weigh 72 grams?

5. If $18\frac{1}{2}$ litres of water fill one bucket, how many buckets of the same capacity will be filled with 129.5 litres of water?

6. A tailor had a fabric roll $49\frac{1}{2}$ metres long. If the tailor cut the roll into 11 pieces of the same length, what was the length of each piece?

7. Maize flour weighing $72862\frac{1}{2}$ kilograms was packed into packets of 50.25 kilograms each. How many packets were used?

8. A rope 279 metres long was cut into pieces of $15\frac{1}{2}$ metres each. How many pieces of the rope were obtained?

9. A tailor makes a pair of shorts by using a cloth $1\frac{3}{4}$ metres long. How many pairs of shorts can be made by a tailor from a fabric of length $10\frac{1}{2}$ metres?

10. How many $\frac{1}{1000}$ are there in $\frac{1}{10}$?

11. A share obtained after dividing $\frac{13}{6}$ by x is $\frac{2}{3}$. Find the value of x.

12. A $\frac{75}{100}$ metres long was cut into equal parts each $\frac{125}{1000}$ metres long. How many pieces were obtained?

13. Henry feeds his chickens with maize bran of $1\frac{1}{4}$ kilograms every day. How many days will $7\frac{1}{2}$ kilograms of maize bran be used to feed the chickens?

14. Rice weighing $2362\frac{1}{2}$ kilograms was packed in small bags. Each bag could carry 15.75 kilograms of rice. How many bags were used.

15. Angela walked a distance of $162\frac{1}{2}$ metres. If one step had a length of 0.65 metres, how many steps did she walk?

16. Zuberi divided $\frac{1}{2}$ of his farm equally to all of his children. If each child got $\frac{1}{8}$ of the farm, how many children got the portions of the farm?

17. How many $\frac{75}{100}$ are there in 60?

18. In a certain village, families were provided with 2 tons of maize. If each family got $\frac{1}{20}$ of one ton, how many families are there in the village?

19. A basketball team practises for $17\frac{1}{2}$ hours every week. If the players use a total of $192\frac{1}{2}$ hours in practice, how many weeks did the team practise?

20. A school bought $359\frac{5}{8}$ metres of a fabric roll for tailoring school uniforms. If each pupil needs $2\frac{5}{8}$ metres, how many pupils will get uniforms?

Revision exercise

Answer the following questions:

1. $12 \div \frac{1}{6} =$
2. $305 \div \frac{5}{12} =$
3. $1\ 100 \div 1\frac{7}{8} =$
4. $84 \div 5\frac{1}{4} =$

5. $2\,754 \div \frac{5508}{75688} =$

6. $16\frac{5}{7} \div 65 =$

7. $\frac{147}{720} \div 98 =$

8. $\frac{121}{1000} \div 22 =$

9. $\frac{18}{67500} \div 9 =$

10. $\frac{93}{1818} \div 31 =$

11. $\frac{2045}{8} \div 50 =$

12. $\frac{6}{7} \div \frac{3}{28} =$

13. $\frac{3}{7} \div \frac{999}{56280} =$

14. $\frac{7}{5} \div \frac{70}{1651} =$

15. $\frac{2}{35} \div \frac{7035}{500} =$

16. $56\frac{1}{4} \div \frac{30}{16} =$

17. $16\frac{2}{3} \div 8\frac{1}{3} =$

18. $\frac{25}{100} \div 0.5 =$

19. $\frac{18}{1000} \div 0.03 =$

20. $\frac{4}{5} \div 0.25 =$

21. $\frac{25}{24} \div 3.75 =$

22. $\frac{60}{1024} \div 0.625 =$

23. Thread 315 centimetres long was cut into equal pieces of $4\frac{1}{2}$ centimetres each. How many pieces of the thread were obtained?

24. Groundnuts weighing $90\frac{1}{2}$ kilograms were divided equally into five packages. Find the mass of each package.

25. A fabric roll has a length of $49\frac{1}{2}$ metres. If the roll is cut into 11 equal pieces, find the length of each piece.

26. How many $\frac{2}{3}$ are there in $13\frac{1}{3}$?

27. How many pieces of wood of length $1\frac{1}{4}$ metres can be obtained from the wood of length $9\frac{7}{8}$ metres?

28. How many 0.125 are there in $\frac{1}{4}$?

29. A parking area has $472\frac{1}{2}$ square metres. If one car occupies 7.5 square metres, how many cars of the same size can be parked in the area?

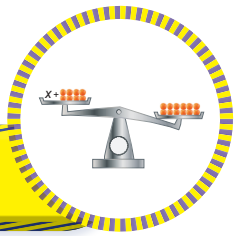
30. A lorry carried packets of flour each weighing 97.5 kilograms. If all the packets weighed $682\frac{1}{2}$ kilograms, how many packets did the lorry carry?

Summary

1. When dividing whole numbers by fractions or fractions by whole numbers, first, convert whole numbers to fractions by dividing them by 1.
2. When dividing fractions by decimals, first, convert decimals into fractions.
3. When dividing fractions by fractions, multiply the dividend by the reciprocal of the divisor. The answer obtained should be expressed in the simplest form.
4. Dividing fractions by fractions is the same as writing
$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$; $b \neq 0$, $c \neq 0$ and $d \neq 0$.

Chapter Nine

Algebra



Introduction

In Standard Five and Six, you learnt algebraic expressions and how to use mathematical operations to simplify them. In this chapter, you will learn to simplify algebraic expressions and solve word problems involving algebra. The competencies gained will enable you to simplify algebraic expressions and solve equations formed from real life situations.

Exercise 1: Revision

Answer the following questions:

1. Simplify the following algebraic expressions:

(a) $3x + 4x - x =$

(b) $18v - 2v + v - 21v =$

(c) $\frac{s}{2} + s + \frac{1}{3}s =$

(d) $5\frac{1}{2}t - \frac{16}{5}t =$

(e) $21xy \div 7y =$

(f) $16p^2q^3 \div 4pq^2 =$

(g) $a(0.6a + 3a) =$

(h) $15y^6 \div 3y^5 =$

(i) $\frac{4e - e}{3e} =$

(j) $\frac{2}{3}k + \frac{5}{6}k - \frac{4}{6}k =$

2. Find the value of the unknown variables in the following questions:

(a) $-2 + 5y = 13$

(b) $8x = 2x + 18$

(c) $3 + 5p = 38 - 2p$

(d) $10m + 15 - 55 + 6m - 24 = 0$

(e) $\frac{2}{7}r = 2$

(f) $\frac{1}{2} + 2x = 2\frac{1}{2} + x$

(g) $q - 0.05 = 85 - 2q$

(h) $0.8x - 0.8 = 1.6$

(i) $\frac{4}{5}y = 4 - \frac{1}{5}y$

(j) $\frac{3}{4}f - \frac{2}{4}f - \frac{1}{6} = \frac{1}{8}f$

Simplification of algebraic expressions

Algebraic expressions involve the operations of addition, subtraction, multiplication or division. Algebraic expressions are simplified by performing addition or subtraction of like terms. Algebraic expressions with like variables and exponents are called like terms. For example, 8y and 19y or 8yx and 3xy or $6x^2y$ and $2x^2y$ are like terms. Algebraic expressions with unlike variables and exponents are called unlike terms. For example, 6x and 6z or 3mn and 4vy or $5u^2y$ and $6y^2v$ are unlike terms. Terms can have coefficients which are whole numbers, fractions, decimals or mixed numbers. Examples of algebraic expressions having terms with whole numbers are: $3x + 5x + 15y$ and $4m$. Examples of algebraic expressions having terms with fractions are: $\frac{1}{3}x + \frac{2}{3}x$ and $\frac{1}{2}y + 2$. Examples of algebraic expressions having terms with decimals are: $0.4x + 0.5x + 1.5y$ and $0.6m - 2.5m$. Examples of algebraic expressions having terms with mixed numbers are: $4\frac{1}{3}x + \frac{2}{3}x + 0.6y$ and $4x + \frac{2}{5}x + 9y - 0.7$.

Addition and subtraction of algebraic expressions with whole number coefficients

Example 1

Simplify: $12xy + 7xz^2 + 2(5xy - xz^2)$.

Solution

Open brackets:

$12xy + 7xz^2 + 2(5xy - xz^2) = 12xy + 7xz^2 + 10xy - 2xz^2$.

Collect like terms:

$12xy + 7xz^2 + 10xy - 2xz^2 = 12xy + 10xy + 7xz^2 - 2xz^2$.

Simplify by adding or subtracting like terms:

$12xy + 10xy + 7xz^2 - 2xz^2 = 22xy + 5xz^2$.

Therefore, $12xy + 7xz^2 + 2(5xy - xz^2) = 22xy + 5xz^2$.

Example 2

Simplify: $14y - 6xy + 2y - xy$.

Solution

Collect like terms as follows:

$14y - 6xy + 2y - xy = 14y + 2y - 6xy - xy.$

Simplify by adding or subtracting like terms:

$14y + 2y - 6xy - xy = 16y - 7xy.$

Therefore, $14y - 6xy + 2y - xy = 16y - 7xy.$

Example 3

Simplify: $2y - 3(x + y) + 10x.$

Solution

Open brackets:

$2y - 3(x + y) + 10x = 2y - 3x - 3y + 10x.$

Collect like terms:

$2y - 3x - 3y + 10x = 2y - 3y - 3x + 10x.$

Simplify by adding or subtracting like terms:

$2y - 3y - 3x + 10x = -y + 7x.$

Therefore, $2y - 3(x + y) + 10x = 7x - y.$

Exercise 2

Simplify the following algebraic expressions:

1. $5x + 8y + x + 11y$
2. $6mn + 17q + mn + q$
3. $(k + y) + 2y + (k + y)$
4. $4rs - 12r - 7rs - 8r - 5rs$
5. $12z^2y - 3zy(2z - 4)$
6. $4nm^2 - (3m - 3nm^2) + 4m$
7. $p + q - (p + q) + 2pq$
8. $10uv - 15 + 55 - 6uv + 24$

9. $4a - 3(a - 18)$

10. $8xy - 24x - (7xy - 11x) + 4xy$

11. $24pq + 5st - 8pq + 3st$

12. $4xy + 8wz - 3xy - 8wz + 2xy$

13. $8y^2 + 6x^2 + 2y^2 - 2x^2 + 4$

14. $4a^2b + 10x^2y^2 + 8a^2b - 2x^2y^2 - 4$

15. $3mn^2 - (2m + 4mn^2) + 4m$

16. $4t^3r^2 - 3ab + 6ab - 3t^3r^2$

17. $5m^2p - 4(m^2p + 2mp^2) - 8mp^2$

18. $6ty^2 + 4t^2y - 8ty^2 - 6ty^2 + 8$

19. $a^2b - 4a(ab + a) - 6a^2 + 5$

20. $15mnr - 2kl - 18mnr + 14kl$

Multiplication and division of algebraic expressions with whole number coefficients

When multiplying or dividing the same variables, the governing rules can be deduced. Algebraic expressions with unlike variables are called unlike terms. Variables of unlike terms cannot be simplified when multiplied or divided.

Example 1

Simplify: $b \times b$.

Solution

$b \times b = b^1 \times b^1$

Since they have the same bases add exponents as follows:

$b \times b = b^{1+1}$

$= b^2$.

Therefore, $b \times b = b^2$.

Example 2

Simplify: $y^2 \times y^3$.

Solution

Since the bases are the same, add the exponents as follows:

$y^2 \times y^3 = (y^1 \times y^1) \times (y^1 \times y^1 \times y^1)$

$= y^{(1+1)+(1+1+1)}$

$= y^{2+3}$

$= y^5$.

Therefore, $y^2 \times y^3 = y^5$.

Using Examples 1 and 2, the following rule is deduced:

$$a^m \times a^n = a^{m+n}$$

Example 3

Simplify: $2m \times n \times 3p$.

Solution

Expand the given algebraic expression as follows:

$2m \times n \times 3p = 2 \times 3 \times m \times n \times p$.

Multiply the coefficients and simplify:

$2 \times 3 \times m \times n \times p = 6mnp$.

Therefore, $2m \times n \times 3p = 6mnp$.

Example 4

Simplify: $a^3 \div a^2$.

Solution

$a^3 \div a^2 = \frac{a^3}{a^2}$

$= \frac{a \times a \times a}{a \times a}$

$= a.$

Therefore, $a^3 \div a^2 = a$.

Using Example 4, the following rule is deduced:

$$a^m \div a^n = a^{m-n} \text{ or } \frac{a^m}{a^n} = a^{m-n}$$

Example 5

Simplify: $7xy^3 \times 10x^4y$.

Solution

Expand the given algebraic expression as follows:

$7xy^3 \times 10x^4y = 7 \times xy^3 \times 10 \times x^4y$

$= 7 \times 10 \times xy^3 \times x^4y.$

Multiply the coefficients and variables:

$7 \times 10 \times xy^3 \times x^4y = 70 \times x^{(1+4)}y^{(3+1)}$

$= 70x^5y^4.$

Therefore, $7xy^3 \times 10x^4y = 70x^5y^4$.

Example 6

Simplify: $4p^2t \div 2pt$.

Solution

Write the algebraic expression in fraction: $4p^2t \div 2pt = \frac{4p^2t}{2pt}$

Expand $\frac{4p^2t}{2pt}$ as follows:

$\frac{4p^2t}{2pt} = \frac{4 \times p \times p \times t}{2 \times p \times t}$.

Collect like terms:

$\frac{4 \times p \times p \times t}{2 \times p \times t} = \frac{4}{2} \times \frac{p^2}{p} \times \frac{t}{t}$.

Simplify the algebraic expression obtained:

$\frac{4}{2} \times \frac{p^2}{p} \times \frac{t}{t} = 2p$.

Therefore, $4p^2t \div 2pt = 2p$.

Exercise 3

Simplify the following algebraic expressions:

1. $3l^3 \times 6l^2$
2. $k^4 \times k^5 \times k$
3. $r^5h^6 \times 2rh$
4. $5 \times n \times 6 \times 4n$
5. $-21a^3b^2 \div 7ab^2$
6. $-5te(e^8t)$
7. $14tw^9 \div 7tw$
8. $(18b^3 + 2b^2 + 3b) \div 6b$
9. $c^8 \div 4c^7$
10. $(9v^4 \div 3v) \times 3v$
11. $24m^2n^3 \div 8m^2n$
12. $(6a^2 + 2b - 3c^3) \times 4abc$
13. $72k^8n^7 \div 9k^4n^6$
14. $-5cd(c^4d^2)$
15. $s^{10}t^4u^5 \times 6s^5t^4u^3$
16. $(16n^4r^2 \div 8nr) \times 3nr$
17. $\frac{1}{4}(24x^2y^3 \div 12xy^2)$
18. $(22k^4l^6 \div 11k^2l^3) \times kl$

19. $k^4n^2 \times k^5n^3 \times k^9n^6 \times 4$

20. $144x^8y^2z^4 \div 72x^6yz^2$

Simplification of algebraic expressions with fractional coefficients

Algebraic expressions with fractional coefficients can be simplified as normal fractions. As it is done in simplification of algebraic expressions with whole number coefficients, like terms are collected together and simplified to get a single term. For example; $\frac{2}{3}x$, $\frac{1}{2}xy$, $\frac{21}{50}x$, $\frac{3}{5}x^2y$ are terms with fractional coefficients. The algebraic expressions $\frac{2}{3}x$ and $\frac{21}{50}x$ are examples of like terms, where as $\frac{1}{2}xy$ and $\frac{3}{5}x^2y$ are example of unlike terms.

Addition and subtraction of algebraic expressions with fractional coefficients

Example 1

Simplify: $\frac{2}{3}y + \frac{3}{4}y$.

Solution

Since the variable is the same, add the coefficients:
The LCM of 3 and 4 is 12.

Thus, $\frac{2}{3}y + \frac{3}{4}y = \frac{2y}{3} + \frac{3y}{4}$

$= \frac{(4 \times 2y) + (3 \times 3y)}{12}$

$= \frac{8y + 9y}{12}$

$= \frac{17y}{12}$

$= 1\frac{5}{12}y.$

Therefore, $\frac{2}{3}y + \frac{3}{4}y = 1\frac{5}{12}y.$

Example 2

Simplify: $\frac{1}{2a} + \frac{2}{3a}$.

Solution

Add the coefficients of the given algebraic expression as follows:
The LCM of 2a and 3a is 6a.
Add and simplify the answer:

Thus, $\frac{1}{2a} + \frac{2}{3a} = \frac{(3 \times 1) + (2 \times 2)}{6a}$

$= \frac{3 + 4}{6a}$

$= \frac{7}{6a}$.

Therefore, $\frac{1}{2a} + \frac{2}{3a} = \frac{7}{6a}$.

Example 3

Simplify: $\frac{2x}{5t^2} - \frac{x}{t^2}$.

Solution

Subtract the coefficients as shown:
The LCM of $5t^2$ and t^2 is $5t^2$

Thus, $\frac{2x}{5t^2} - \frac{x}{t^2} = \frac{(1 \times 2x) - (5 \times x)}{5t^2}$

$= \frac{2x - 5x}{5t^2}$

$= \frac{-3x}{5t^2}$.

Therefore, $\frac{2x}{5t^2} - \frac{x}{t^2} = \frac{-3x}{5t^2}$.

Exercise 4

Simplify the following algebraic expressions:

1. $\frac{2}{5}x + \frac{7}{10}x$
2. $\frac{1}{p} + \frac{1}{q}$
3. $t + \frac{t}{8}$
4. $\frac{3y^2}{bc} + \frac{5cy^2}{bc^2}$
5. $\frac{2}{21}q - \frac{q}{7}$
6. $3b - \frac{4}{7}b$
7. $\frac{2}{18t} - \frac{1}{54t}$
8. $\frac{w}{4} + \frac{5w}{8} - \frac{w}{6}$
9. $\frac{3}{5}(c^2 - cd) - \frac{2}{5}c\,(d + c)$
10. $3m - \frac{2m}{3}$
11. $\frac{6y}{7m^2} - \frac{2y}{5m^2}$
12. $\frac{4}{5}(k^2 + ky) - \frac{3}{5}k(y - k)$
13. $\frac{3d^2}{xy} + \frac{4yd^2}{xy^2} - \frac{4yd^2}{xy^2}$
14. $\frac{5m^2}{14t^2} + \frac{m^2}{56t^2} - \frac{3m^2}{56t^2}$
15. $4n^2r - \frac{3n^2r}{7}$
16. $\frac{2}{15}kt - \frac{kt}{3} + \frac{kt}{4}$
17. $\frac{1}{4}(3 - 5n) - \frac{3}{4}(5 + 4n) + \frac{1}{3}$
18. $\frac{kl}{12} + \frac{2kl}{5} - \frac{5kl}{6}$
19. $\frac{5}{9}(m^2n - mn^2) - \frac{2}{9}(m + n)mn$
20. $\frac{2x^2y^2}{13} + \frac{x^2y^2}{26} - \frac{5x^2y^2}{52}$

Multiplication and division of algebraic expressions with fractional coefficients

Example 1

Simplify: $\frac{1}{2}a \times \frac{3}{5}b$.

Solution

Expand the algebraic expression:

$\frac{1}{2}a \times \frac{3}{5}b = \frac{1}{2} \times a \times \frac{3}{5} \times b.$

Collect the like terms and multiply:

$\frac{1}{2} \times a \times \frac{3}{5} \times b = \frac{1}{2} \times \frac{3}{5} \times a \times b$

$= \frac{1 \times 3}{2 \times 5} \times ab$

$= \frac{3}{10} ab.$

Therefore, $\frac{1}{2}a \times \frac{3}{5}b = \frac{3}{10}ab.$

Example 2

Simplify: $\frac{1}{2}m \div \frac{1}{4}m.$

Solution

$\frac{1}{2}m \div \frac{1}{4}m$ can be written as $\frac{m}{2} \div \frac{m}{4}$

Using the rule $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ then simplify:

$\frac{m}{2} \div \frac{m}{4} = \frac{m}{2} \times \frac{4}{m}$

$= \frac{4m}{2m} = 2.$

Therefore, $\frac{1}{2}m \div \frac{1}{4}m = 2.$

Example 3

Simplify: $\frac{15}{21}a^3c \div \frac{1}{14}ac^3.$

Solution

$\frac{15}{21}a^3c \div \frac{1}{14}ac^3 = \frac{15a^3c}{21} \div \frac{ac^3}{14}.$

Apply the rule of fraction division:

$\frac{15a^3c}{21} \div \frac{ac^3}{14} = \frac{15a^3c}{21} \times \frac{14}{ac^3}$.

Collect like terms and simplify:

$$\frac{15a^3c}{21} \times \frac{14}{ac^3} = \frac{15}{21} \times 14 \times \frac{a^3c}{ac^3}$$

$$= 10 \times \frac{a^2}{c^2}$$

$$= \frac{10a^2}{c^2}.$$

Therefore, $\frac{15}{21} a^3c \div \frac{1}{14} ac^3 = \frac{10a^2}{c^2}$.

Exercise 5

Simplify the following algebraic expressions:

1. $\frac{9}{f} \div \frac{1}{f}$
2. $\frac{8}{9}p^2l \div \frac{12}{27}pl$
3. $\frac{1}{6}mn^2 \times \frac{3n}{9m^3n}$
4. $\frac{5}{9}t^2q^3 \times \frac{3}{5}b^2t^2$
5. $6a^2b \times \frac{2}{3}b^3$
6. $\frac{3}{7}x^3y^4 \div x^2y$
7. $\frac{2}{3}a^3b^3 \div \frac{3}{4}ab^2$
8. $\frac{1}{2}m^4 \div \left(\frac{m}{2}\right)^3$
9. $\frac{5r^3}{12h^2} \times \left(\frac{6y^2}{3r} \div \frac{5y}{4h^3}\right)$
10. $\frac{5}{3}(a^3c + a^2c) \div \frac{1}{3}(a^2c)$
11. $\frac{14}{k^2l^3} \div \frac{m^3n^5}{56}$
12. $\frac{13}{28}m^4n^6 \div \frac{m^3n^5}{56}$
13. $8p^3q^2 \times \frac{3}{4}pq^3$
14. $\frac{3}{8}l^5m^2 \div \frac{5}{8}l^2m$
15. $\frac{2t^2}{5n^3}\left(\frac{2k^4}{3t} \div \frac{2k^2}{5n^4}\right)$
16. $\frac{1}{7}a^{10}b^4 \div \frac{3}{49}a^5b^2$
17. $\frac{1}{2}m^7n^2l^6 \times \frac{1}{4}m^9n^5l^4$
18. $\frac{6}{11}l^3j^3k^5 \times \frac{11}{36}l^5jk^3$

19. $\frac{1}{8}a^2(3a^4b \div a^6b^2)$

20. $\left(\frac{4zn}{5}\right)^3 \div \frac{4zn}{75}$

Simplification of algebraic expressions with decimal coefficients

Algebraic expressions with decimal coefficients can be simplified by addition, subtraction, multiplication and division of the coefficients. As it is done in simplification of algebraic expressions with whole numbers and fractional coefficients, terms with decimal coefficients which can be added or subtracted are like terms only. Also, like variables in a term can be multiplied or divided.

Addition and subtraction of algebraic expressions with decimal coefficients

When adding or subtracting algebraic expressions with decimal coefficients, consider the like variables in the given terms. For example, $0.6x + 0.3y$ and $0.6x - 0.3y$ cannot be simplified because the terms have unlike variables.

Example 1

Simplify: $0.3x + 0.32x$.

Solution

These terms have like variables. Therefore, the terms can be added. Add the coefficients using a vertical method by considering the place value of the digits:

$$\begin{array}{r} 0.30 \\ +\ 0.32 \\ \hline 0.62 \end{array}$$

Multiply the sum of coefficients by the given variable.
$0.62 \times x = 0.62x$.

Therefore, $0.3x + 0.32x = 0.62x$.

Example 2

Simplify: $23.4ab^2 - 5.2ab^2$.

Solution

Subtract the coefficients using a vertical method by considering the place value of the digits:

$$\begin{array}{r} 2\ 3.4 \\ -\quad 5.2 \\ \hline 1\ 8.2 \\ \hline \end{array}$$

Multiply the sum of coefficients by the given variable:
$18.2 \times ab^2 = 18.2ab^2$.

Therefore, $23.4ab^2 - 5.2ab^2 = 18.2ab^2$.

Exercise 6

Simplify the following algebraic expressions:

1. $4.13y + 6.524y$
2. $0.016rs + 7.68rs$
3. $173.1c^3d^2 + 29.7c^3d^2$
4. $578.83t + 3748.149t$
5. $3.6u + 2.72u + 31.9u$
6. $5.04m - 0.97m$
7. $40.2abc - 11.88abc$
8. $25.01g^2h - 14.13g^2h$
9. $83.45p^3 - 2(1.7p^3 - 8.345p^3)$
10. $61.7n + 4.05n - 18.86n$
11. $7.56t^2r^2 - 3(1.7t^2r^2 - 3.4t^2r^2)$
12. $5(8.7a^2b + 9.2a^2b) - 0.18a^2b$
13. $295.72k^4l^3 - 2kl(k^3l^2 + 5.43k^3l^2)$
14. $65.5n^2r^4 - 58.2n^2r^4 + 0.15n^2r^4$
15. $19.87ab(a^2 + b) - 12.92a^3b - 2.45ab^2$
16. $85.671m^2a - 64.924m^2a + 4(3.21m^2a)$
17. $0.048t^2u^2 - 4tu(0.98tu - 1.59tu)$
18. $5.75y^2a - 2.5(3ay^2 + 2ay^2)$
19. $2s(240.4sp - 125.3sp + 81.8sp)$
20. $169ts(2.4t^2s - 1.6ts^2) - 14.7t^2s^2(4t + 3s)$

Multiplication and division of algebraic expressions with decimal coefficients

Recall that, decimal numbers are multiplied as whole numbers, decimal places are counted and placed in the answer. When multiplying algebraic expressions with decimal coefficients, multiply coefficients of the terms, and then multiply the variables using the rule of multiplication of exponents. Also, when dividing terms with decimal coefficients, start dividing the coefficients, and then divide the variables by using the rule of dividing exponents.

Example 1

Simplify: $0.2y^3 \times 0.5y^4$.

Solution

Multiply the coefficients and the variables as shown:

$$\begin{aligned} 0.2y^3 \times 0.5y^4 &= 0.2 \times y^3 \times 0.5 \times y^4 \\ &= 0.2 \times 0.5 \times y^3 \times y^4 \\ &= 0.1 \times y^{3+4} \\ &= 0.1y^7. \end{aligned}$$

Therefore, $0.2y^3 \times 0.5y^4 = 0.1y^7$.

Example 2

Simplify: $1.2xy^2 \times 0.3x^3y$.

Solution

Multiply the coefficients and the variables as shown:

$$\begin{aligned} 1.2xy^2 \times 0.3x^3y &= 1.2 \times x \times y^2 \times 0.3 \times x^3 \times y \\ &= 1.2 \times 0.3 \times x \times x^3 \times y^2 \times y \\ &= 0.36 \times x^4 \times y^3 \\ &= 0.36x^4y^3. \end{aligned}$$

Therefore, $1.2xy^2 \times 0.3x^3y = 0.36x^4y^3$.

Example 3

Simplify: $6.9b^4 \div 2.3b^3$

Solution

$6.9b^4 \div 2.3b^3 = \frac{6.9b^4}{2.3b^3}$

$= \frac{6.9 \times b \times b \times b \times b}{2.3 \times b \times b \times b}$

$= 3b.$

Alternative solution

$6.9b^4 \div 2.3b^3 = \frac{6.9}{2.3}\frac{b^4}{b^3}$

$= 3b^{4-3}$

$= 3b.$

Therefore, $6.9b^4 \div 2.3b^3 = 3b.$

Exercise 7

Simplify the following algebraic expressions:

1. $m^2 \times m$
2. $k \times k \times k \times 0.1k^3$
3. $1.44y \times 14.4y$
4. $0.35t \times 0.4t^8$
5. $2.6x \times 3.2x \times 10.5x^6$
6. $0.125ab^5 \times 0.8a^2b$
7. $0.25n^3 \times 0.25n^3 \times n^3$
8. $0.5dc^2 \times 0.2d^2c \times 1.5c^4d^3$
9. $18.62a^3b^2c \times 2.2cba$
10. $700.5xy \times 0.001xyz$
11. $4.8x^8 \div 0.1x^5$
12. $t^3 \div 0.5t$
13. $3.2z^5 \div 0.4z^4$
14. $2.8a^3 \div 0.07a$
15. $0.6b^2 \div 0.25a^2$
16. $4.5t^4 \div 5t^3$
17. $2.8x^3y^2 \div 1.4x^2y$
18. $0.6m^2n \div 0.75mn^2$
19. $\frac{4.8x^9y^6z}{0.12x^8y}$
20. $\frac{31.0p^2}{15.5r} \div \frac{p}{2.5r^3}$

Finding the value of algebraic expressions

You can find the value of an algebraic expression by substituting the value of a given variable, and then perform the mathematical operations in the algebraic expression.

Example 1

Find the value of $\frac{10ab}{c}$, if a = 2, b = 3 and c = 5.

Solution

$\frac{10ab}{c} = \frac{10 \times a \times b}{c}$.

Substitute the values of a, b and c into the algebraic expression:

$\frac{10ab}{c} = \frac{10 \times 2 \times 3}{5} = 12$.

Therefore, the answer is 12.

Example 2

If n = 3, r = 4 and t = −3, find the value of $(n + r) \times t$.

Solution

Substitute the values of n, r and t into the given algebraic expression:

$(n + r) \times t = (3 + 4) \times (-3)$

$= 7 \times (-3)$

$= -21$.

Therefore, the answer is −21.

Example 3

Find the value of $y^m + ym^b$, given that y = 4, b = 3 and m = 2.

Solution

Substitute the values of y, b and m into the given algebraic expression:

$y^m + y m^b = 4^2 + 4 \times 2^3$

$= 16 + 4 \times 8$

$= 16 + 32$

$= 48.$

Therefore, the answer is 48.

Exercise 8

1. Given that t = 12, r = 4 and p = 4, find the value of the following algebraic expressions:

(a) $\frac{t - 2r}{p}$

(b) $t^2 + r^2 - p^2$

(c) $(t + r - p) - (r + p)$

(d) $\frac{8t}{3(rp - t)}$

(e) $\frac{t}{r}(t \div p)$

2. If m = 12, n = 10, r = 6, π = 3.14, find the value of the following algebraic expressions:

(a) $2(m + n)$

(b) $(m \div r) - \frac{(mn)}{r}$

(c) πr^2

(d) $\sqrt{(n + r) - m}$

(e) $5(n^2r - 3nr)$

(f) $3\pi\left(\frac{m^2 - r^2}{m^2 + n^2}\right)$

(g) $4\pi m\left(\frac{n - r}{n + r}\right)$

3. Find the value of the following algebraic expressions, if a = 5, b = 2 and c = 4:

(a) $\frac{5}{12}\left(\frac{a^2 - c^2}{a^2 + b^2}\right)$

(b) $(6a - 3b) + \frac{144}{3c}$

(c) $b\left(\frac{a^2 - cb}{2a}\right)$

(d) $\frac{4a + 2b - 3c}{2a + b + c}$

(e) $2cb(3a^2 \div 5b)$

(f) $6c^3b^2(a^2 \div 15b - 3c)$

(g) $\frac{a^3 + 2b^4 - 3c^2}{2abc}$

(h) $\frac{-a^2 + b^2}{(a + b)(a + b)}$

Word problems on algebra

Example 1

Jawala's present age is x years. If the age of her mother is three times her age, find her mother's age.

Solution

Jawala's age is x years

Her mother's age is (Jawala's age) $\times$ 3

$= x \times 3$

$= 3x.$

Therefore, her mother's age is 3x years.

Example 2

The sum of three consecutive odd numbers is 57. Find the three numbers.

Solution

Recall that consecutive odd numbers differ by 2. If the first number is y, the second number will be (y + 2), and the third number will be (y + 4).

So, the three consecutive numbers are y, (y + 2) and (y + 4).

The sum of these numbers is 57.

Formulate an equation as follows:

$y + (y + 2) + (y + 4) = 57$

$3y + 6 = 57$

$3y + 6 - 6 = 57 - 6$

$3y = 51.$

Divide both sides by 3:

$$\frac{3y}{3} = \frac{51}{3}$$

$y = 17.$

Thus, y = 17, y + 2 = 17 + 2 = 19, y + 4 = 17 + 4 = 21.

Therefore, the first number is 17, the second number is 19 and the third number is 21.

Exercise 9

Answer the following questions:

1. Tumaini is x years old. What will be his age after 10 years?

2. The sum of two numbers is 100. If one of the numbers is t, what is the second number?

3. A class has a total of w pupils. If the number of girls is twice the number of boys, find the number of boys.

4. A piece of wood has a length of $5x + 3$ centimetres. If a piece $x - 10$ centimetres long was cut and removed. Find the length of the remaining wood.

5. A rectangular farm has a length of 9n centimetres and width of 5m centimetres. Find:
   (a) The perimeter of the farm.
   (b) The area of the farm.

6. Find the cost of buying 12 kg of sugar if the cost of 1 kg is y shillings.

7. The weight of 20 similar pencils is z grams. Find the weight of one pencil.

8. Rushaka bought a hoe at $p - q$ shillings, machete at $p + q$ shillings and sickle at $p - 3q$ shillings. How much money did he spend to buy the items altogether?

9. Which number when multiplied by $\frac{1}{6}$, then 5 is subtracted from the product, the answer is 3?

10. Find the area of a right angled triangle whose height is 3x centimetres and base is 4x centimetres.

11. A father's age is nine times the age of his child. After three years, the father's age will be five times the child's age. Find their present age.

12. The sum of three consecutive even numbers is 42. Find the three numbers.

13. When 55 is added to a certain number and the sum is divided by 3, the result is four times the number. Find the number.

14. The sum of one fifth of an even number and one sixth of a consecutive even number is 15. Find the two numbers.

15. Asha's age is y years:
(a) What was her age 3 years ago?
(b) What will be her age after 4 years?
(c) Find her age if half of her age 3 years ago is equal to one third of her age after 4 years.

16. A Mother's age is 37 years and that of her daughter is 5 years. How long should it take for the mother's age to be five times her daughter's age?

17. If the angles of a triangle are $(5x - 14)°$, $(2x + 2)°$, and $(x + 32)°$. Find the value of x.

18. A garden's width is 20 percent of its length. If the perimeter of the garden is 39.6 metres, Find:
(a) The length and width of the garden.
(b) The area of the garden.

19. Edah received a loan from a bank. She gave her daughter a quarter of the loan. Also, she gave her son one fifth of the loan. If she remained with 110 000 shillings. Find:
(a) The amount of money she gave her daughter.
(b) The total amount of loan received from the bank.

20. The sum of two consecutive numbers is 21. Find the two numbers.

Revision exercise

Answer the following question:

1. From the following list of terms identify the like terms:

$4x^2, x^3, -x^2y, 5x^3, 4x^2y, 6x^2, -7x^2, -2x^2y, -3xy^2, -x^3$

2. Simplify the following algebraic expressions:

(a) $2x - 3x + 5x + x$
(b) $5y^2n^4 - 3y^2n^4 - 2yn$
(c) $2v^2 + 3u^2 - 2v^2 - 4u^2 + v + vu$
(d) $t^3 + tk + 2t^3 - 4tk + 6tk - t^3$
(e) $2a^3b \times 4ab^2$
(f) $18a^2b^3 \times 9a^3$
(g) $m^2n \times nm \times n^3 \times m^2$
(h) $3p^3q^3 \div p^3q$
(i) $2l^5 \times 3l^4 \div 4l^2$
(j) $8p - 4p \div 2p + 3p$
(k) $(m + y) - y + n - (m + y)$
(l) $-\frac{5}{9}x^2y + \frac{3}{5}x^2y + x^2y$
(m) $p - \frac{2}{5}p + \frac{1}{2}p$
(n) $\frac{1}{2}m - \frac{1}{4}m - \frac{1}{8}m$
(o) $2\frac{1}{7}y - y + \frac{2}{5}y$
(p) $\frac{3}{5}(c^2 - cy) + \frac{3}{5}cy$
(q) $\frac{5z}{12y} \times \frac{6z}{3x} \div \frac{5z}{4y}$
(r) $4.13x + 6.524x + 12.186x$
(s) $0.006y + 100.12y + 1001.11y$
(t) $4.25f - 2.3f - 0.75f$
(u) $40.2ab^2 - 11.88ab^2$
(v) $mn - 0.7mn - 0.125mn$
(w) $\frac{1}{6}x^2 + \frac{1}{2}x^2 + \frac{1}{3}x^2$
(x) $\frac{7}{9}ab + \frac{2}{3}ab - \frac{3}{2}ab$
(y) $0.11p^4q + 0.5pq^3 - 0.2p^4q + pq^3$

3. If l = 4, b = 3, h = 2, π = 3.14 and r = 5 find the value of the following algebraic expressions:

(a) $\pi r^2 h$

(b) $2(l + b)$

(c) $\frac{4}{3}\pi r^2$

(d) $\frac{\pi l}{h}\left(\frac{r^2 - h^2}{r^2 + h^2}\right)$

(e) $\frac{\pi r}{h}(0.62b - 0.45l)$

4. Given a = 2, b = 1, c = 3, find the value of the following algebraic expressions:

(a) $\frac{1}{3}c^2(a^2 - b^2)$

(b) $c^a + a^c$

(c) $\sqrt{c^3 - a^2 + 2b}$

5. Abeid bought a total of 5 chickens. He bought x chickens each at 5 000 shillings and the remaining chickens each at 6 000 shillings. How much did he spend to buy all chickens?

6. A fruit seller bought y number of pawpaws at the market of which, 20 pawpaws were spoiled along the way. If the seller sold the unspoiled pawpaws at 1 500 shillings each, find the total amount of money the seller earned.

7. Kabanza spent 500 shillings more to buy pineapples than the amount of money he spent to buy mangoes. He also bought avocados for twice the amount he spent to buy mangoes. If he used y shillings to buy mangoes, find the total amount of money he spent.

8. The school farm has a total of $4p^7$ orange seedlings. $2p^2$ pupils need to equally weed the seedlings. How many orange seedlings will each pupil weed?

Summary

1. Algebraic expressions which can be simplified by addition or subtraction are those with like terms. Terms are simplified by adding or subtracting their coefficients.
2. Terms which are multiplied can be simplified by multiplying the coefficients, and then followed by multiplying the variables.
3. In order to multiply or divide algebraic terms, multiply or divide the variables. Also, multiply or divide the coefficients.
4. In order to add like algebraic terms, add the coefficients. Also, in order to subtract like terms, subtract the coefficients.

Chapter Ten

Speed

Introduction

In this chapter, you will learn the concept of speed, formula for speed, and how to use the formula to calculate distance, time, and speed of various objects. Also, you will be able to learn about the speed of different bodies or objects moving on the land, in water, and in the atmosphere. The competencies gained will help you to understand schedules of vehicles leaving or arriving at their stations, the uses of a speedometer (Figure 1) for example in motorbikes, cars, and various vehicles.

The concept of speed

When two cars start a journey at the same time and cover the same distance, they can reach their final destinations at different times. If the cars do not stop until they reach their final destinations, the one with higher speed will arrive first. Likewise, in athletics, a winner runs at a high speed to cover the required distance in a short time. The distance covered per time taken when travelling, running, walking or swimming is called speed.

The speed of moving bodies differs from one another. For example, an aeroplane moves at a higher speed than a car. Also, the speeds of some cars are higher than those of some motorbikes, and a motorbike moves at a higher speed than a bicycle. Aeroplanes, cars and motorbikes are some of the means of transport which use speedometers for measuring their speed. Figure 1 shows a speedometer used for measuring speed in kilometres per hour.

Figure 1: *A speedometer*

Some animals and birds can move faster than human beings. For example, a cheetah can run at a speed of more than 113 kilometres per hour. Figure 2 shows a cheetah running at a high speed.

Figure 2: *A cheetah running at a high speed*

An international standard football ground is approximated to have a perimeter of 0.328 kilometres. This means that a cheetah can run around the football ground 345 times per hour. That is, $\frac{113 \text{ kilometres}}{0.328 \text{ kilometres}}$ per hour = 345 times per hour.

An eagle has the ability of flying at a speed of up to 320 kilometres per hour. This means that the eagle can fly around a football ground 975 times per hour. That is, $\frac{320 \text{ kilometres}}{0.328 \text{ kilometres}}$ per hour = 975 times per hour. Figure 3 shows an eagle flying at a high speed.

Figure 3: *An eagle flying at a high speed*

An athlete can run up to a distance of 37 kilometres per hour. This means that an athlete can run around the football ground 113 times per hour. That is,

$\frac{37 \text{ kilometres}}{0.328 \text{ kilometres}}$ per hour = 113 times per hour.

Figure 4 shows an athlete running around a field.

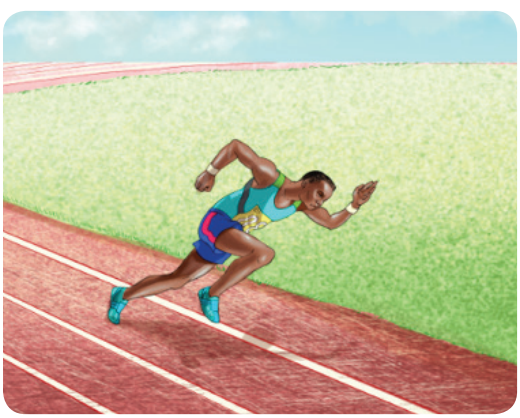


Figure 4: *An athlete running around a field*

Formula for speed

Speed is the measure of distance moved in a given time. The time can be an hour, a minute or second. Therefore, the formula for speed is obtained by dividing the distance covered by the time taken. Thus,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

From the speed formula, distance and time can be described as follows:
Distance is the length between two objects or points. The Standard International (SI) unit of distance is metre abbreviated as 'm'. Other units of distance include millimetre (mm), centimetre (cm), decimetre (dm), decametre (dam), hectometre (hm) and kilometre (km). The devices used for measuring distance can be tape measures and rulers.

Time refers to hours, minutes or seconds spent in moving from one point to another. The SI unit of time is second abbreviated as 's'. Other units used in measuring time include hours and minutes. A tool used for measuring time is called clock or watch. The stopwatch is used to measure the starting time and ending time of a certain event. The stopwatch can have a circular shape with minute and second arrows. Also, it can be a digital stopwatch which shows hours, minutes and seconds. Figure 5 and Figure 6 show different types of stop watches.

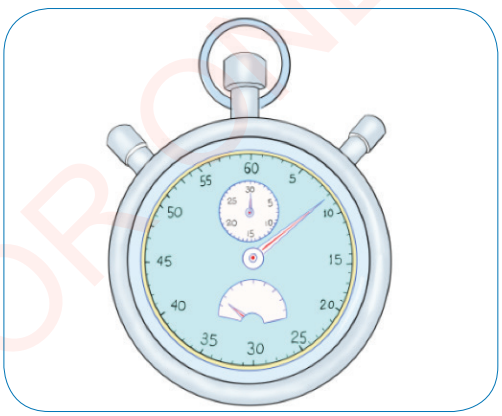


Figure 5: *An analogue stopwatch*

Figure 6: A digital stopwatch

The unit of speed is obtained by dividing the unit of distance by the unit of time. For example, $\frac{\text{metre}}{\text{second}}$ or $\frac{\text{kilometre}}{\text{hour}}$. However, the SI unit of speed is $\frac{\text{metre}}{\text{second}}$ abbreviated as m/s.

If the values of two quantities among the distance, time and speed are known, the value of the other quantity can be obtained. The following table shows metric units and their symbols for distance, time and speed used in calculations.

Distance	Time	Speed	Abbreviation
Kilometre (km)	Hour (hr)	Kilometre per hour	km/hr
Metre (m)	Hour (hr)	Metre per hour	m/hr
Metre (m)	Second (s)	Metre per second	m/s
Centimetre (cm)	Second (s)	Centimetre per second	cm/s

Activity 1: Calculating the speed using distance and time by using graph paper

Do this activity with your fellow pupils by following your teacher's instructions.

Example 1

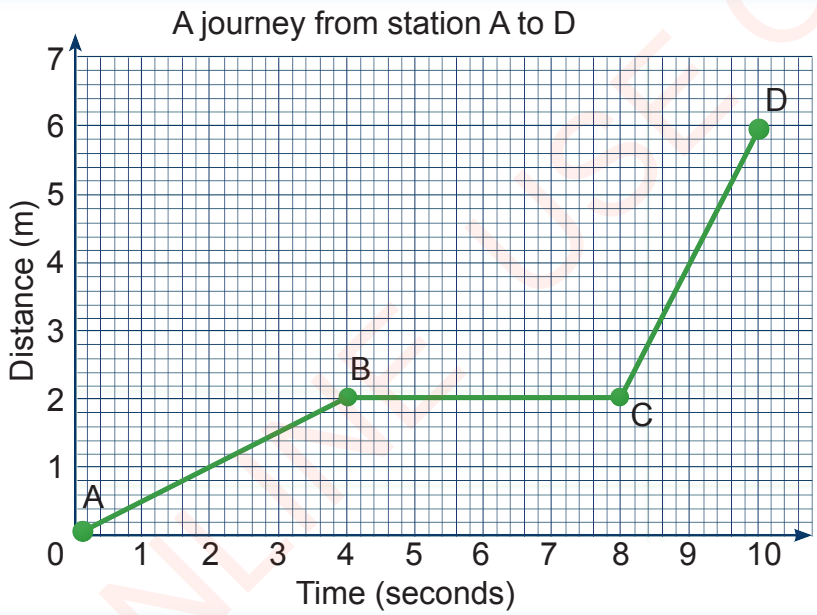
Jane and Juma live in the same household and they both study at Juhudi Primary School. Juma spends 40 minutes to walk from home to school. Jane spends 30 minutes to walk from home to school using the same route. Who walks at a higher speed between the two?

Solution

Jane and Juma walk the same distance.
Since the distance is the same, the person who spends a shorter time will have a higher speed. Jane spends 30 minutes and Juma spends 40 minutes. Thus, Jane spends a shorter time.
Therefore, Jane walks at a higher speed than Juma.

Example 2

The following graph shows the distance covered from point A to D against the time taken. Study the graph carefully, and then answer the questions that follow.



(a) What is the time taken to move from point A to B?
(b) What is the distance from point A to B?
(c) What is the time taken to move from point B to C?
(d) What is the distance from point B to C? Explain the meaning of the distance obtained.
(e) What is the time taken to move from point C to D?
(f) What is the distance from point C to D?
(g) Which part has a higher slope between moving from point A to B and C to D?

Solution

(a) The horizontal axis represents the time taken. Thus, the time taken from point A to B = The horizontal coordinate of point B minus horizontal coordinate of point A. That is,
Time taken from A to B = 4 seconds – 0 second = 4 seconds.
Therefore, the time taken to move from point A to B is 4 seconds.

(b) The distance from point A to B = The vertical coordinate of point B minus the vertical coordinate of point A. That is,
Distance from A to B = 2 metres – 0 metres
= 2 metres.
Therefore, the distance from point A to B is 2 metres.

(c) The time taken from B to C = 8 seconds – 4 seconds
= 4 seconds
Therefore, the time taken from point B to C is 4 seconds.

(d) The distance from B to C = 2 metres – 2 metres
= 0 metres
Therefore, the distance from point B to C is 0 metres. This means that, there is no movement between the points.

(e) The time taken from C to D = 10 seconds – 8 seconds
= 2 seconds
Therefore, the time taken from point C to D is 2 seconds.

(f) The distance from C to D = 6 metres – 2 metres
= 4 metres
Therefore, the distance from point C to D is 4 metres.

(g) The slope between the points A and B

$$= \frac{\text{Vertical increase from point A to B}}{\text{Horizontal increase from point A to B}}$$

$$= \frac{2m - 0m}{4s - 0s} = \frac{2m}{4s} = 0.5 \text{ m/s}.$$

The slope between the points C and D is 0.5 $^{m}/_{s}$.

The slope between the points C and D

$$= \frac{\text{Vertical increase from point C to D}}{\text{Horizontal increase from point C to D}}$$

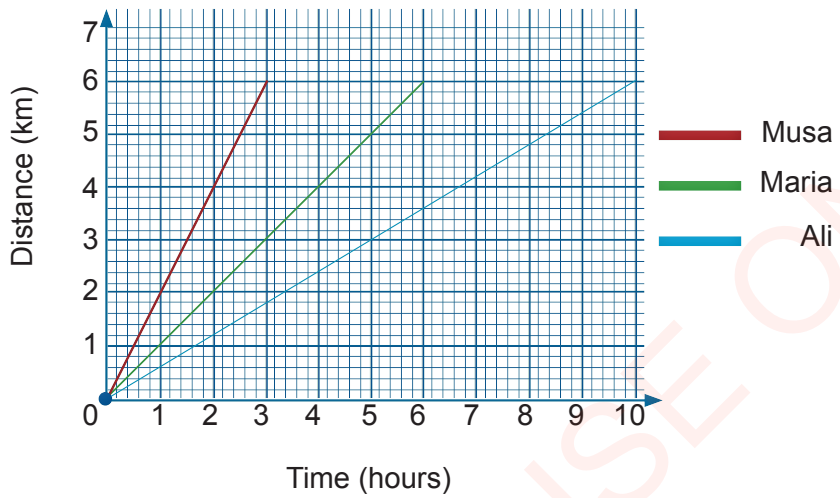
$$= \frac{6m - 2m}{10s - 8s} = \frac{4m}{2s} = 2 \ ^{m}/_{s}.$$

The slope between the points C and D is 2 $^{m}/_{s}$.

Therefore, the slope from C to D is higher than that from A to B.

Example 3

Carefully study the following graph showing the speed of three athletes, and then answer the questions that follow.



(a) Which athlete had the highest speed? Give reasons.
(b) Which athlete used longer time than others?
(c) What distance did each athlete run?

Solution

(a) The graph shows that all the athletes ran a distance of 6 kilometres. Musa spent 3 hours, Mary used 6 hours and Ali used 10 hours. Therefore, Musa ran at the highest speed because he covered the distance in a shorter time than others.
(b) Ali spent more time to run the same distance of 6 kilometres.
(c) Each athlete ran a distance of 6 kilometres.

Activity 2: Measuring the distance and time used in running

Do this activity with your fellow pupils by following the instructions given by your teacher.

Computation of speed

Given distance and time, speed can be computed by using the following formula:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}.$$

Example 1

A car travels a distance of 480 kilometres in 6 hours. Find its speed.

Solution

Apply the formula for speed:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

From the question, identify the distance and the time taken. That is,
Distance = 480 kilometres
Time = 6 hours
Substitute the values of distance and time in the formula to get the speed of the car. That is,

$$\text{Speed} = \frac{480 \text{ km}}{6 \text{ hours}} = 80 \text{ km / hr.}$$

Therefore, the speed of the car is 80 kilometres per hour.

Example 2

Edina ran 70 metres in 10 seconds. What was her speed?

Solution

Apply the formula for speed:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

From the question, identify the distance and the time taken. That is,

Distance = 70 metres
Time = 10 seconds
Substitute the values of distance and time in the formula to get the speed of Edina. That is,

$$\text{Speed} = \frac{70 \text{ m}}{10 \text{ s}}$$

$$= 7 \text{ m / s}$$

Therefore, Edina's speed was 7 metres per second.

Example 3

A car travelled a distance of 459 000 metres in 9 000 seconds. Find the speed of the car in kilometres per hour.

Solution

Use the following steps:
Apply the formula for speed:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Convert the given metric units of distance in metres into kilometres and time in seconds into hours:

(i) Distance: 459 000 metres = 459 kilometres

(ii) Time: 9 000 seconds = $2\frac{1}{2}$ hours

Substitute the values of distance and time in the formula to get the speed of the car. That is,

$$\text{Speed} = \frac{459 \text{ km}}{2\frac{1}{2} \text{ hr}}$$

$$\text{Speed} = \frac{459 \text{ km}}{\frac{5}{2} \text{ hr}}$$

$$= 183.6 \text{ km / hr.}$$

Therefore, the speed of the car is 183.6 kilometres per hour.

Example 4

Makungu lives at Mondo Village. Everyday, he rides his motorbike to travel a distance of 15 kilometres in 0.75 hours from the village to school. At what speed does he ride the motorbike?

Solution

Apply the formula for speed:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

From the question, identify the distance and time taken. That is,
Distance = 15 kilometres
Time = 0.75 hours

Substitute the values of distance and time in the formula to get the riding speed. That is,

$$\text{Speed} = \frac{15 \text{ km}}{0.75 \text{ hr}} = \frac{15 \text{ km} \times 100}{75 \text{ hr}}$$

$$= 20 \text{ km / hr.}$$

Therefore, Makungu rides the motorbike at a speed of 20 kilometres per hour.

Exercise 1

Answer the following questions:

1. A tourist car spent 9 900 seconds to travel a distance of 540 000 metres on the road. Find its speed in kilometres per hour.

2. Roza takes 30 minutes to drive her car from home to school and covers a distance of 20 kilometres. At what speed in kilometres per hour does she drive her car?

3. Find the speed of a lorry which used 8 hours to travel a distance of 480 kilometres.

4. A ball was kicked and rolled a distance of 150 metres in 30 seconds. Find its speed in centimetres per second.

5. Jamhuri Primary School prepared to host a 400-metre running tournament. The winner spent 5 minutes. Compute the speed of the winner in metres per second.

6. Basheka spent 2 hours to ride his bicycle a distance of 60 kilometres. Find the speed at which he rode the bicycle in metres per second.

7. Find the speed of a bus that travelled a distance of 230 kilometres in 5 hours.

8. A cat chased a rat for a distance of 20 metres before catching it. Find its speed if it used 0.4 minutes to catch the rat.

9. Salum measured the speed of a spider. He discovered that the spider walked a distance of 30 centimetres in 6 seconds. Find the speed of the spider.

10. Stefano kicked a ball that landed after 3 seconds at a distance of 40 metres. Find the speed of the ball:
    (a) In kilometres per hour.
    (b) In metres per second.

Computation of distance

The formula for speed can be used to find distance if speed and time are known. Figure 7 shows the relationship between distance as a dividend, and speed and time as divisors.

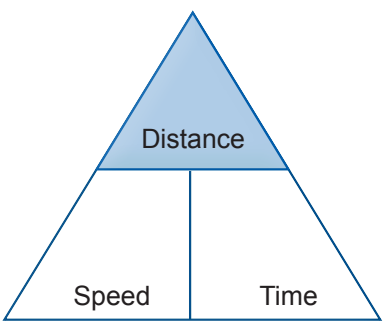


Figure 7: *Distance as a dividend, speed and time as divisors*

From Figure 7, a formula for distance can be deduced as follows:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Cross multiplication in the formula for speed gives:

$$\text{Distance} = \text{Speed} \times \text{Time}$$

Example 1

Majaliwa rode a motorbike at a speed of 24 kilometres per hour. If he continuously rode the motorbike for 4 hours, what distance did he travel?

Solution

Use the following steps:

1. Apply the formula for distance:
   Distance = Speed × Time
2. From the question, identify the values of time and speed. That is,
   Time = 4 hours
   Speed = 24 kilometres per hour
3. Substitute in the formula the values of time and speed to get the distance. That is,

$$\text{Distance} = \frac{24 \text{ km}}{\text{hr}} \times 4 \text{ hr}$$
$$= 96 \text{ km}.$$

Therefore, Majaliwa travelled a distance of 96 kilometres.

Example 2

A lorry travelled for 3 hours at a speed of 78 kilometres per hour. Also, it travelled for 2 hours at a speed of 84 kilometres per hour. Find the total distance covered.

Solution

Use the following steps:

1. Apply the formula for distance:
   Distance = Speed × Time.

2. From the question, identify the values of time and speed for both journeys. That is,
   First part of the journey
   Time = 3 hours
   Speed = 78 kilometres per hour.

   Second part of the journey
   Time = 2 hours
   Speed = 84 kilometres per hour.

3. Substitute the values of speed and time in the formula for both journeys to obtain the distance. That is,
   First part of the journey
   Distance = Speed × Time

   $\text{Distance} = \frac{78 \text{ km}}{\text{hr}} \times 3 \text{ hr}$

   = 234 km.

   Second part of the journey
   Distance = Speed × Time

   $\text{Distance} = \frac{84 \text{ km}}{\text{hr}} \times 2 \text{ hr}$

   = 168 km.

   Total distance = Distance of first part of the journey + Distance of second part of the journey
   = 234 km + 168 km
   = 402 km.

Therefore, the lorry travelled a total distance of 402 kilometres.

Exercise 2

Answer the following questions:

1. If a car travels at a speed of 112 kilometres per hour, what distance will it cover in 3 hours?

2. An athlete ran at a speed of 12.8 kilometres per hour. If he ran for $1\frac{1}{2}$ hours, what distance did he cover?

3. Find the distance covered by an object after $1\frac{1}{2}$ hours of moving at a speed of 67.2 kilometres per hour.

4. A child crawled a distance of 6 metres in one minute. What distance did the child crawl in 50 seconds?

5. A snail glided at a speed of 7 metres per hour. Find the distance it covered after $3\frac{1}{2}$ hours.

6. Musa drove his car for 3 hours at a speed of 70 kilometres per hour. Aisha drove her car a distance of 230 kilometres in 4 hours.
   (a) Who drove for a longer distance in 3 hours?
   (b) What distance would Musa travel in 3 hours at Aisha's speed?

7. Samson is a lorry driver who transports goods to different regions. One day, he drove the lorry at a speed of 80 kilometres per hour for $3\frac{1}{2}$ hours. After resting, he drove a distance of 128 kilometres before the lorry got a breakdown. After maintenance, he drove for 30 minutes at a speed of 96 kilometres per hour. Find the total distance covered.

8. Sofia rode a motorbike for 4.75 hours at a speed of 64 kilometres per hour. What distance did she cover?

9. A whale swam for 17 minutes at a speed of 8 metres per second. What distance did it swim?

10. The speed of sound is 340 metres per second. Juma loudly called Asha who was near an obstacle and an echo was heard. At what distance was Asha if Juma heard the echo after 6.2 seconds?

Computation of time

You can simply recall the formula for time using Figure 7. The figure is used to derive the formula for distance.

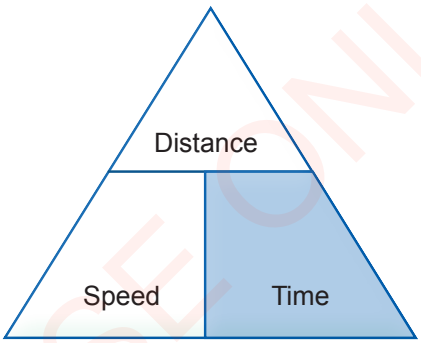


Figure 8: *A formula for time*

From Figure 8, you can observe that the distance is the dividend while speed is the divisor.

Therefore, Time = $\frac{\text{Distance}}{\text{Speed}}$.

Example 1

The distance between the first and second train stations is 96 kilometres. The speed of a train from the first to the second station is 48 kilometres per hour. What time will the train take to arrive at the second station?

Solution

Use the following steps:

1. Apply the formula for time:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}.$$

2. From the question, identify the values of distance and speed. That is,
   Distance = 96 kilometres
   Speed = 48 kilometres per hour.
3. Substitute the values of distance and speed in the formula:

$$\text{Time} = \frac{96 \text{ km}}{48 \text{ km / hr}}$$

$$= 2 \text{ hr.}$$

Therefore, the train used 2 hours to arrive at the second station.

Example 2

A cockroach crawled a distance of 108 centimetres at a speed of 9 centimetres per second. What time did it use to cover the distance?

Solution

Use the following steps:

1. Apply the formula for time:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}.$$

2. From the question, identify the values of distance and speed. That is, Distance = 108 centimetres
   Speed = 9 centimetres per second
3. Substitute the values of distance and speed in the formula:

$$\text{Time} = \frac{108 \text{ cm}}{9 \text{ cm / s}}$$

$$= 12 \text{ s.}$$

Therefore, the cockroach used 12 seconds.

Example 3

A passenger train travelled from Dar es Salaam to Tabora at 64 kilometres per hour. A goods train left Tabora to Dar es Salaam at a speed of 80 kilometres per hour. Both trains left the stations at the same time. If the distance from Tabora to Dar es Salaam is 864 kilometres, how long did it take for the two trains to meet?

Solution

Steps

1. Write the formula of finding distance
   Distance = Speed × Time
2. Total distance = 864 kilometres.
   Distance travelled by the first train = 64 kilometres per hour × time
   Distance travelled by the second train = 80 kilometres per hour × time
   Total distance = distance travelled by the first train + distance travelled by second train. Let t = time
   Total distance = 64 km / hr × t + 80 km / hr × t

$$864 \text{ km} = 144 \text{ km / hr} \times t$$

$$t = \frac{864 \text{ km}}{144 \text{ km / hour}}$$

$$t = 6 \text{ hours.}$$

Therefore, the trains took 6 hours to meet.

Example 4

One car left from Dar es Salaam to Mbeya. An hour later, another car left from the same station to Mbeya at a speed of 80 kilometres per hour. After three hours, the second car overtook the first car. Find the speed of the first car.

Solution

The second car will overtake the first car when they will have travelled equal distance. Distance travelled by the first car = distance travelled by second car. If the second car left an hour after the first car, then from the distance formula, we have:

Distance = Speed × Time

Let t = Time

Speed of the first car = s_1

Speed of the second car = s_2

Distance travelled by the first car = $s_1 \times (1 + t)$

Distance travelled by the second car = $s_2 \times t$.

But distance travelled by the first car is equal to the distance travelled by the second car.

Thus, $s_1 (1 + t) = s_2 \times t$

Since t = 3 hours, then s_1 (1+3) hours = 80 km / hour × 3 hours

$4s_1$ = 240 km / hour

$s_1 = \frac{240 \text{ km}}{4 \text{ hr}}$

$s_1 = 60$ km / hour.

Therefore, the speed of the first car was 60 kilometres per hour.

Exercise 3

Answer the following questions:

1. Deo travelled 36 kilometres at a speed of 8 kilometres per hour. Anna travelled 48 kilometres at a speed of 10 kilometres per hour:
   (a) Who travelled in a shorter time?
   (b) What time did he/she use to travel?

2. How much time will Hamisi take to run 42 kilometres at a speed of 12.8 kilometres per hour?

3. Janet drove her car 72 kilometres from her workplace to her home at a speed of 48 kilometres per hour. Jerome drove his car 81 kilometres from his workplace to his home at a speed of 60 kilometres per hour. If both started the journey at the same time,
   (a) who arrived home earlier than the other?
   (b) what was the difference in their arrival times?

4. How long does it take to drive a car 416 kilometres at a speed of 104 kilometres per hour?

5. Ali is at a distance of 216 kilometres away from Zena. Zena can ride a motorbike towards Ali at a speed of 72 kilometres per hour. What time can she take to arrive where Ali is?

6. Emma travelled 36 kilometres at a speed of 9 kilometres per hour. Elia travelled 50 kilometres at a speed of 10 kilometres per hour:
   (a) Who arrived earlier than the other?
   (b) Find the difference of their arrival times.

7. How long does it take to travel a distance of 624 kilometres at a speed of 96 kilometres per hour?

8. Idrisa runs 2 metres per second. How long will he take to cover $1\frac{1}{2}$ metres?

9. A train travels at 50 metres per second. Find the time it will take to travel a distance of 10 000 kilometres.

10. An athlete runs a distance of 30 kilometres in 60 minutes. How long will the athlete take to run 20 kilometres at the same speed?

Exercise 4

Read carefully the following statements, and then write **True** for correct statements and **False** for incorrect statements in the space provided:

1. Speed is calculated by dividing the distance covered by time taken .....

2. The speed of a chameleon goes up to 480 kilometres per hour .....

3. A car can travel at a speed of 96 kilometres per hour or higher .....

4. Time = Speed × Distance .....

5. Distance = Speed × Time .....

6. The metric units of time are seconds, days, minutes, weeks, years and hours .....

7. The metric units of distance are kilometre, metre, centimetre, week, millimetre and decimetre .....

8. To convert speed in metres per second into kilometres per hour, multiply by $\frac{5}{18}$ .....

9. To convert speed in kilometres per hour into metres per second, multiply by $\frac{5}{18}$ .....

10. Distance = $\frac{\text{Time}}{\text{Speed}}$ .....

Revision exercise

Answer the following questions:

1. Write the formula for calculating speed.

2. Mention any two metric units of speed which are commonly used.

3. Some tourists canoed for two days at a speed of 20 kilometres per hour. What distance did they cover?

4. How long will it take to run 100 metres at a speed of 5 metres per second?

5. A train travelled a distance of 300 kilometres for 4 hours. Find its speed in kilometres per hour.

6. A snail glided 400 centimetres in one hour. Find its speed in:
   (a) Metres per hour.
   (b) Centimetres per minute.

7. Andrea's school is located 1.5 kilometres from his home. Andrea leaves his home at 6:30 am, and arrives at school at 7:20 am. Find the speed he uses when going to school.

8. Milka takes $2\frac{1}{2}$ hours to drive her car for a journey of 40 kilometres and 500 metres. Find the speed of her car in kilometres per hour.

9. Zainabu rides a bicycle at a distance of 32 kilometres for 2 hours and 30 minutes. Find the speed she rides in metres per minute.

10. Hussein, Hidaya and Yakobo are bus drivers. They drive their buses everyday from their village to town at a distance of 96 kilometres. Hussein uses an average of $1\frac{1}{2}$ hours, Hidaya uses an average of $2\frac{1}{2}$ hours, and Yakobo uses an average of $1\frac{3}{4}$ hours. Find the average speed of each driver.

11. Rebeka used her car to travel a distance of 562 kilometres for two days. On the first day, she drove 344 kilometres for 4 hours before resting.

(a) Find the speed at which she drove her car on the first day.

(b) If she used the same speed, find the time used to travel on the second day.

12. Juma took 10 hours to drive his car at a distance of 976 kilometres.

(a) What speed did he use?

(b) If he could drive at a speed of 16 kilometres per hour faster than the previous speed, how much time would he take to cover the same distance?

13. A car travelled 80 kilometres at a speed of 80 000 metres per hour. How much time in minutes did the journey take?

14. Two birds flew westward from a tree. The first bird flew at a speed of 72 kilometres per hour. If the second bird flew at the speed of 48 kilometres per hour, what was the distance between the two birds after 2 hours?

15. Hassan's clock loses 10 seconds in every minute. The clock was initially set at 9.00 am.

(a) What time will it show if the correct clock reads 10:00 am?

(b) Hassan wants to watch a football match at 7:30 pm. What time will his clock show?

16. Juma and Rose were waiting for transport at a bus station. Juma's bus left at 12.35 pm, and Rose left by another bus at 1.05 pm. Both buses were moving in the same direction with the speed of 80 kilometres per hour. Find the distance between the two buses at 2.05 pm.

17. A lorry carrying livestock and a fuel tanker travelled to Dar es Salaam from the same station. The lorry carrying livestock left the station first. After 2 hours, the fuel tanker started the journey at a speed of 45 kilometres per hour. The fuel tanker overtook the lorry carrying livestock after 4 hours. What was the speed of the lorry carrying livestock?

18. Sabina drove a car from her home to town at 40 kilometres per hour. After some time, Jeni drove to town from Sabina's home at a speed of 48 kilometres per hour. Jeni overtook Sabina after 5 hours. What time did Sabina take before she was overtaken by Jeni?

19. Selina travelled by an aeroplane at a distance of 4 512 kilometres. The aeroplane travelled half of the distance at a speed of 1 344 kilometres per hour. If the aeroplane travelled the remaining distance at a speed of 1 330 kilometres per hour, how long did the journey take?

20. Hussein rode a motorbike from school to his home at a speed of 40 kilometres per hour. After one hour his friend rode a motorbike from school towards the opposite direction at a speed of 50 kilometres per hour. Find the time in hours since Hussein left the school if the distance between them was 400 kilometres.

21. An aeroplane travelled for 3.6 hours at a speed of 800 kilometres per hour. It continued travelling for 5 hours and 59 minutes at a speed of 960 kilometres per hour. Find the average speed of the entire journey.

22. A passenger train left the station 2 hours before a goods train started the journey. The trains travelled in opposite directions. The goods train travelled at 56 kilometres per hour for nine hours. After a certain time, the distance between the trains was 448 kilometres. Find the speed of the passenger train.

23. Issa rode a motorbike from school to his friend at a speed of 44 kilometres per hour. Noela left the school at the same time and travelled in the opposite direction while moving on a straight road at 55 kilometres per hour. How long did it take for Issa and Noela to move 60 kilometres apart?

24. Amina left the airport to a mountain. After 2 hours, Jose left the airport to the same mountain at a speed of 56 kilometres per hour. Jose and Amina met after 1 hour and 30 minutes. Find Amina's speed.

25. A journey takes 3 hours to go and 4 hours to return. The returning speed is 12 kilometres per hour. Find the speed of going.

26. A passenger aeroplane made a return trip to Dodoma. The plane flew at a speed of 704 kilometres per hour when going. It flew at a speed of 770 per hour on its return. If it flew for 9 hours on its return, how long did it take when going?

27. Abdul drove his car to a factory at 50 kilometres per hour. After a certain time, Zainabu drove her car at 60 kilometres per hour to the factory from the same station. Zainabu met Abdul 6 hours later. How long did Abdul drive before he met Zainabu?

28. Hashim and Fatuma work at the same office. Hashim left the office to his home. Fatuma left the office one hour later at 48 kilometres per hour and met Hashim after 2 hours. Find the speed of Hashim.

29. Julius rides a motorbike at a speed of 50 kilometres per hour. If he increases his speed by twenty percent, find his new speed in metres per minute.

30. A parrot flies at a speed of 64 kilometres per hour. If the parrot increases its speed by fifty percent, how long will it take to fly 528 kilometres?

Summary

1. The relationship between speed, distance and time is given by:

   $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$.

2. The Standard International metric unit of speed is metre per second. The other commonly used unit is kilometre per hour.

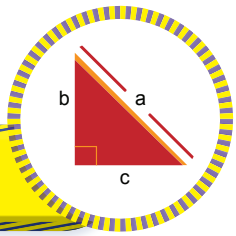
3. Distance and time are calculated using the following formula:

   (a) Distance = Speed $\times$ Time

   (b) $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

Chapter Eleven

Pythagoras' theorem



Introduction

In this chapter, you will learn the relationship between the sides of the right-angled triangle. You will also learn the Pythagoras' theorem and how to use it to find the base, height, and hypotenuse of the right-angled triangle. The competencies gained in this chapter will help you in daily life activities such as architectectural works, construction of buildings, carpentry and shortening travelling distance.

Importance of the Pythagoras' theorem in our daily life

Pythagoras is a name of a Greek mathematician, who proved the relationship between the sides of a right-angled triangle. The importance of Pythagoras' theorem in our daily life can be seen in the following examples:

(a) Shortening travelling distance

The Pythagoras' theorem helps to shorten the route from one place to another. For example, Figure 1 shows a standard seven pupil who lives in street A and studies at a school located in street C. The pupil can use two ways to reach the school. The pupil can use the road from street A via street B. Also, the pupil can go straight to school by using the road from street A to street C. Figure 1 shows that the distance from street A to street C via street B is longer compared to the distance from street A to street C.

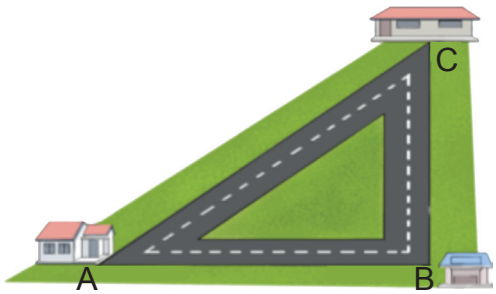


Figure 1: *Uses of Pythagoras' theorem to shorten the travelling distance*

(b) Drawing plans of different buildings

Architects use the Pythagoras' theorem to draw architectural plans of different buildings and other construction works. Such plans include houses, schools, hospitals, industries, airports, roads and bridges. Figure 2 shows a plan of a single storey building.

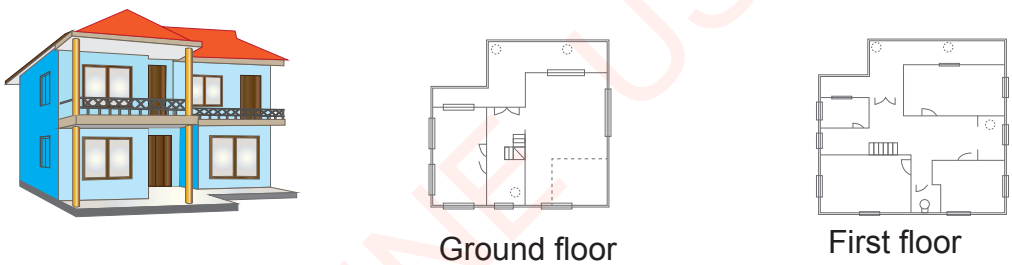


Figure 2: *Single storey building plan*

(c) Drawing maps of different cities

Architects use the Pythagoras' theorem to find the distance and height of different objects when drawing maps of cities. For instance, finding the slope of hills and mountains as shown in Figure 3.

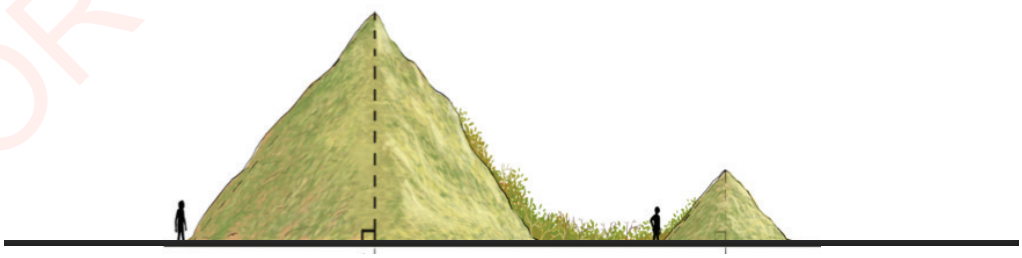


Figure 3: *Slope of a mountain and a hill*

(d) Construction of the foundation of a building with two walls forming a right-angled corner between them

Masons use the Pythagoras' theorem to construct a house foundation whose two walls meet and form a right angle. For instance, the lengths of the three sides of right-angled triangle are 3 metres by 4 metres by 5 metres. These are used by masons to measure the foundation with right-angled walls by using a rope that makes the length of the walls.

Activity: Measuring the right angle in the construction of a house foundation with two walls which make the right angle

In collaboration with your fellow pupils, prepare a rope not less than 15 metres, a protractor, at least three wooden pegs, a ruler, a hammer and a tape measure. Follow the instructions given by the teacher to carry out the activity.

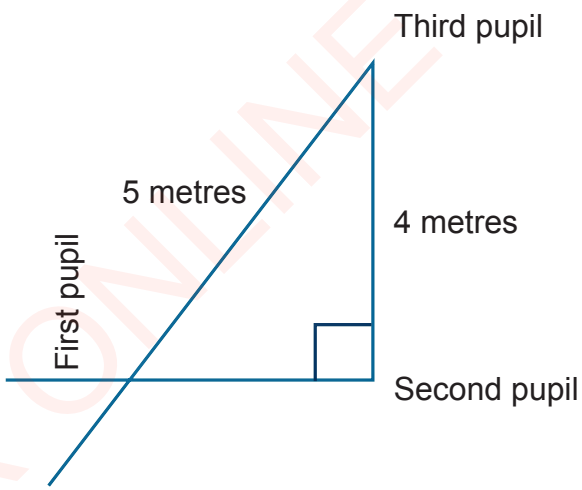


Figure 4: *Uses of the Pythagoras' theorem in the construction of a house foundation with two walls which make a right angle*

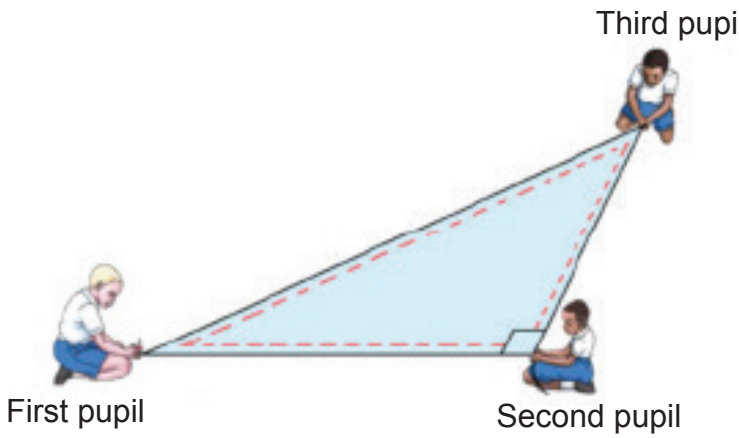


Figure 5: *Measuring a right angle during the construction of a house foundation with two walls which make the right angle*

Recognising the sides of the right-angled triangle

A right-angled triangle has three sides: the base, height and hypotenuse. One of the three angles of the triangle, is a right angle (90 degrees). The right angle is formed by two sides which are the base and height. In Figure 6, the horizontal side of the right-angled triangle is called the base and the vertical side is called the height. The opposite side of the right angle is called the hypotenuse. The hypotenuse is the longest side of all sides. However, the hypotenuse is shorter than the total length of the base and height.

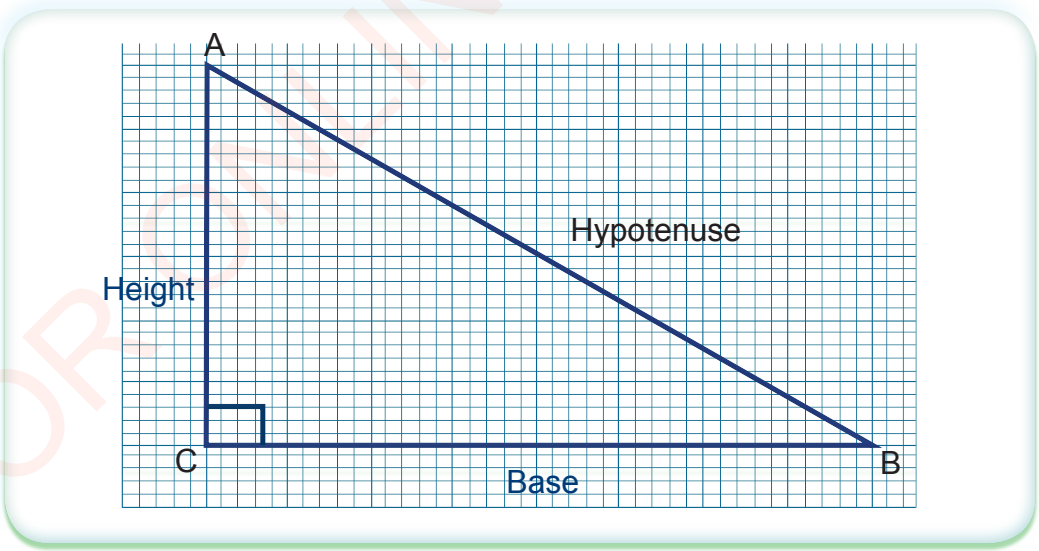


Figure 6: *To identify the base, height and hypotenuse of the right-angled triangle*

Triangle ABC in Figure 7 is the right-angled triangle. Angle ACB is a right angle, that is $A\hat{C}B = 90^{o}$.

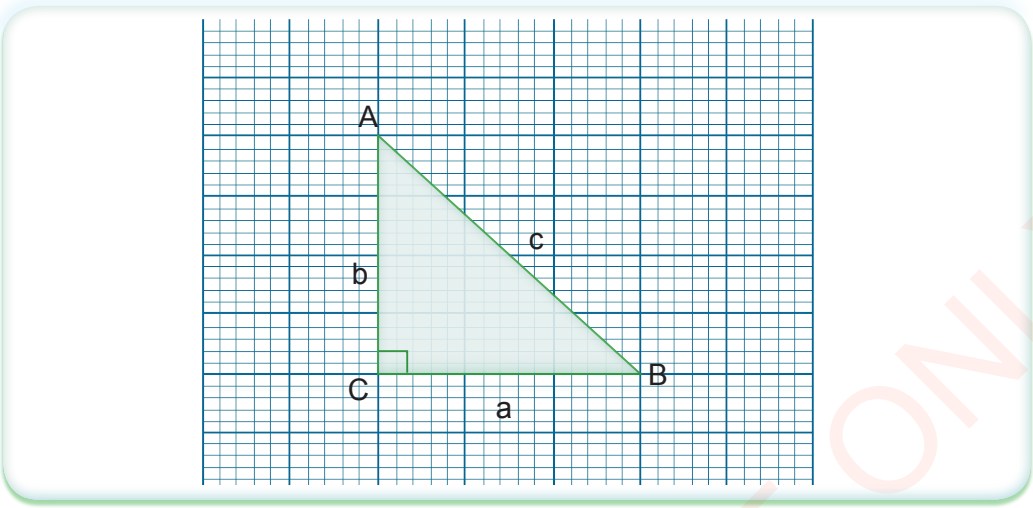
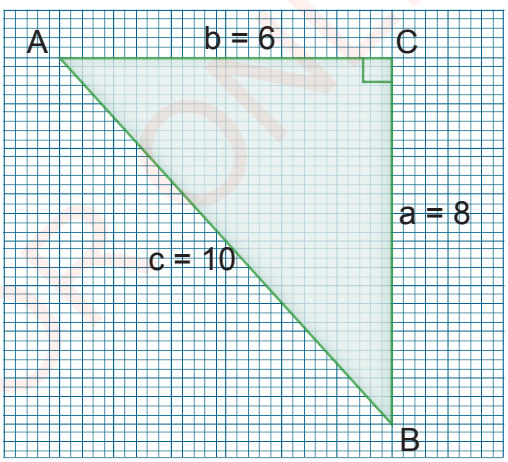
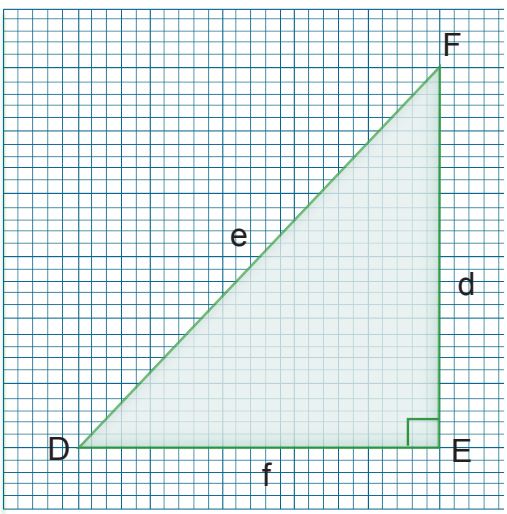


Figure 7: *Showing the length of the base, height and hypotenuse of the right-angled triangle*

$\overline{BA}$ is the longest side of all sides which is called the hypotenuse and its length is c. $\overline{CA}$ is the height whose length is b. $\overline{CB}$ is the base whose length is a.

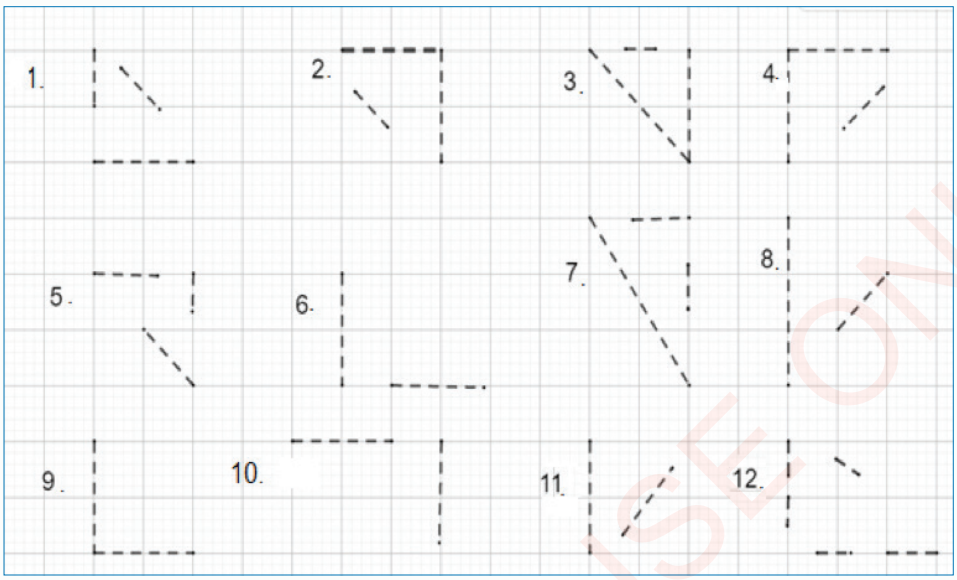
Therefore, the lengths of the right-angled triangle ABC are a, b, and c. The following are examples of right-angled triangles with the base, height, and hypotenuse:





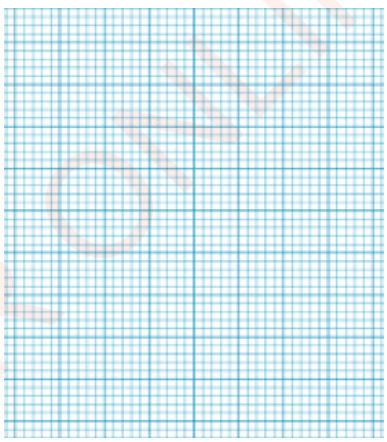
Exercise 1

Draw a right-angled triangle by connecting the dotted lines in the following figures and in each, indicate the base, height and hypotenuse.



In questions 13 and 14, use the graph paper to draw right-angled triangles of different sizes. Show the height, base and hypotenuse.

13.



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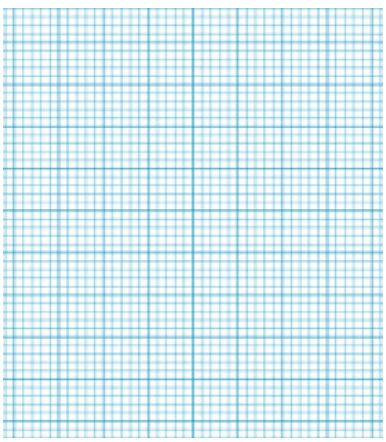


Pythagoras' Theorem

The Pythagoras' theorem is used to find the length of the sides of a right-angled triangle. The theorem states that, "the square of the length of the hypotenuse is equal to the sum of the squares of the length of base and height". That is, the Pythagoras' theorem says, $a^2 + b^2 = c^2$. The theorem is formed by using the right-angled triangle with lengths of the base a, height b, and hypotenuse c as shown in Figure 8.

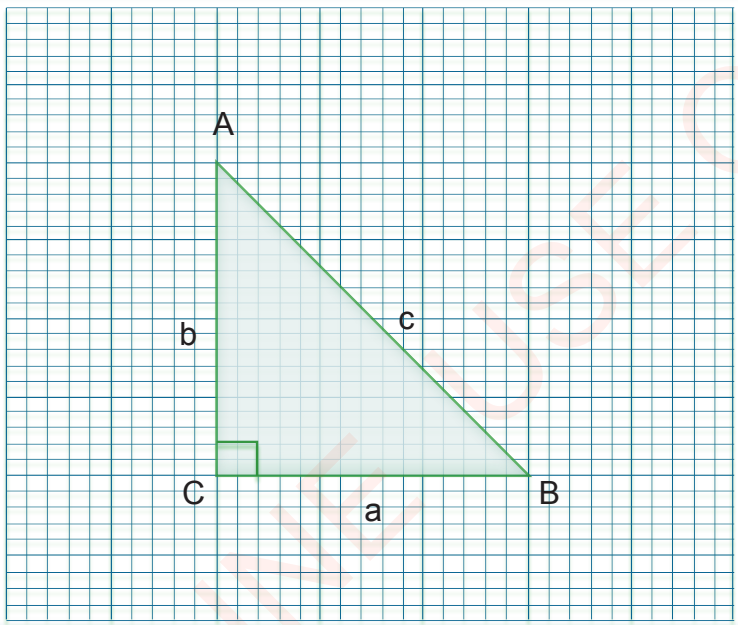


Figure 8: *The lengths of the base, height, and hypotenuse of the right-angled triangle*

Verification of the Pythagoras' theorem

One of the methods used to verify the Pythagoras' theorem is to use the area of squares formed by the sides of the right-angled triangle. The theorem can be verified by counting unit squares for each square formed by the base, height, and hypotenuse. For example, use the right-angled triangle with the base of 4 centimetres, height of 3 centimetres, and hypotenuse of 5 centimetres. Follow the following steps to verify the Pythagoras' theorem:

1. Draw the right-angled triangle with a base of 4 centimetres, height of 3 centimetres, and hypotenuse of 5 centimetres.
2. Make unit squares from the base, height and hypotenuse as shown in Figure 9.
3. Count the number of unit squares resulting from the base of 4 centimetres. The total number of unit squares is 16.
4. Count the number of unit squares resulting from the height of 3 centimetres. The total number of unit squares is 9.
5. Count the number of unit squares resulting from the hypotenuse of 5 centimetres. The total number of unit squares is 25.
6. Find the total number of unit squares of the base and height. The total is $16 + 9 = 25$.
7. Compare the answer in steps 5 and 6. The answers are the same and the total number of unit squares is 25.

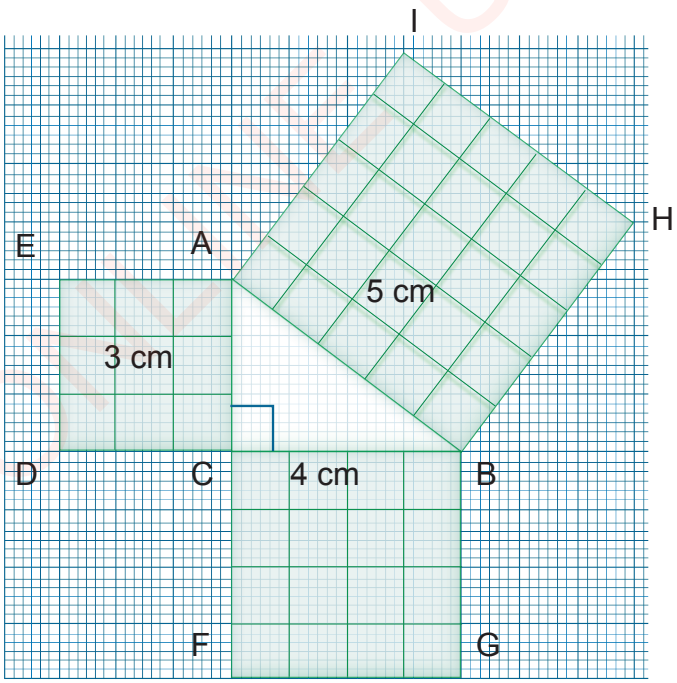


Figure 9: *Verifying the Pythagoras' theorem using area of the squares*

Therefore, the Pythagoras' theorem states that:

(Number of small squares in the base side (The area of the square of the base side)) + (Number of small squares in the height side (The area of the square of the height side)) = (Number of small squares in the hypotenuse side (The area of the square of the hypotenuse side))

Thus, $16 + 9 = 25$ unit squares
But, $16 = 4^2$, $9 = 3^2$ and $25 = 5^2$.

Therefore, $16 + 9 = 25$ unit squares can be written as:
$(4\text{ cm})^2 + (3\text{ cm})^2 = (5\text{ cm})^2$, where 4 cm is the length of base, 3 cm is the the length of height, and 5 cm is the the length of hypotenuse. If a is the the length of the base, b is the the length of the height and c is the the length of the hypotenuse, then $a^2 + b^2 = c^2$.
Therefore, $a^2 + b^2 = c^2$, which is the Pythagoras' theorem.

Figure 10 shows a ladder forming the hypotenuse, a wall forming the height and a floor forming the base. Thus, the ladder, the wall and the floor form the right-angled triangle.

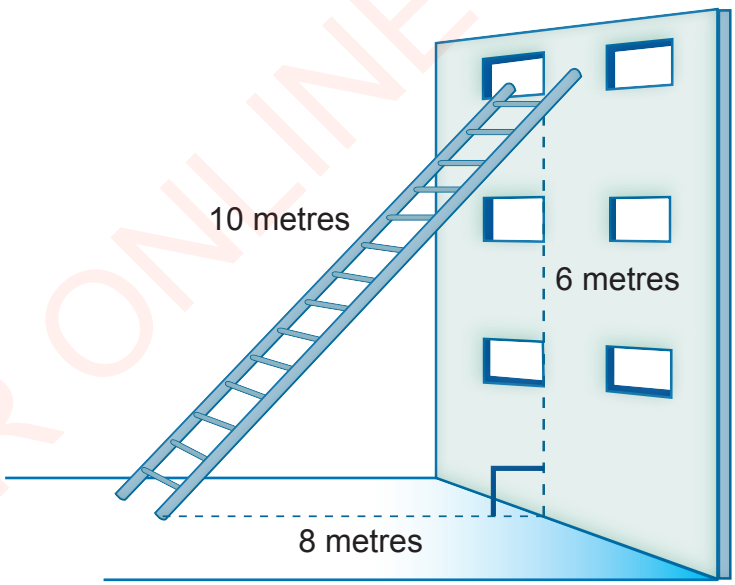


Figure 10: *The ladder forms the hypotenuse side when leaning against the wall and the floor*

Exercise 2

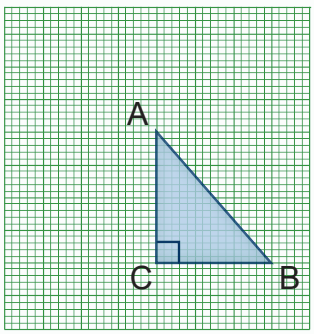
Carefully study the following figures and answer the questions that follow.

(a) Copy all drawings (1-10) on a graph paper and draw the squares as well as the unit squares on the side of the base, height, and hypotenuse.

(b) Count the unit squares resulting from the base, height, and hypotenuse.

(c) What conclusion can you make from the answer obtained in (b) above?

1.



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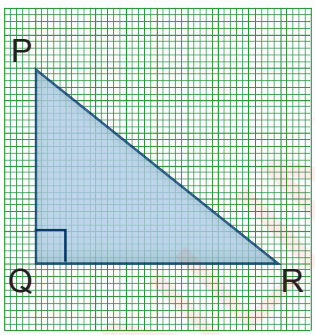

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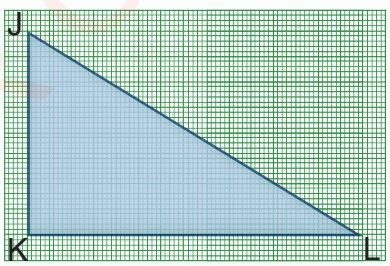

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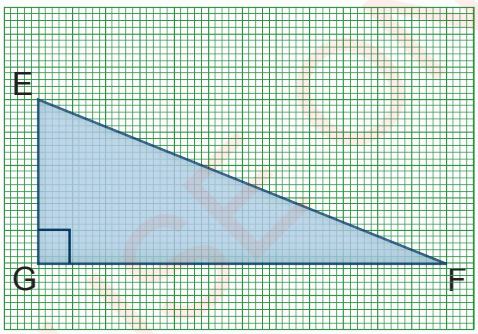

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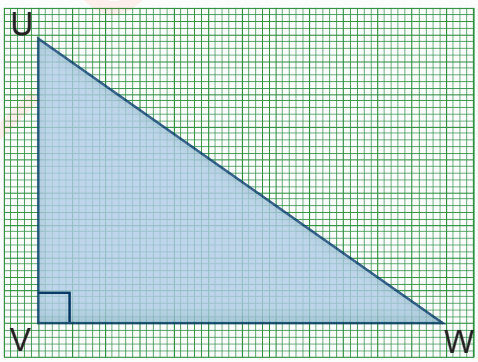

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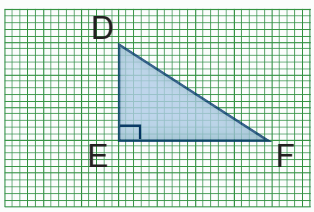

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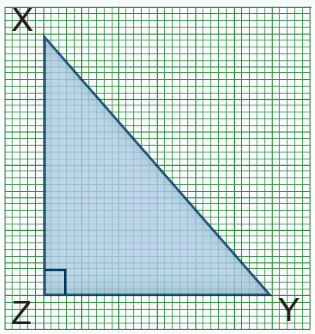

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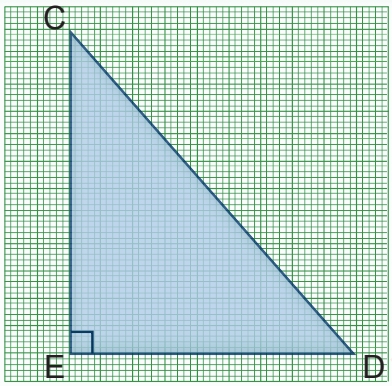

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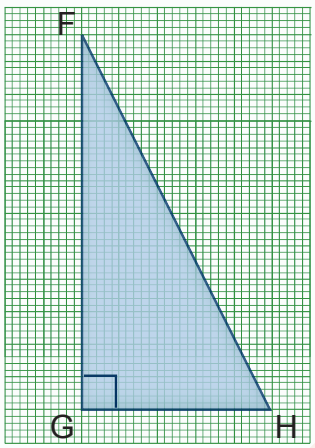

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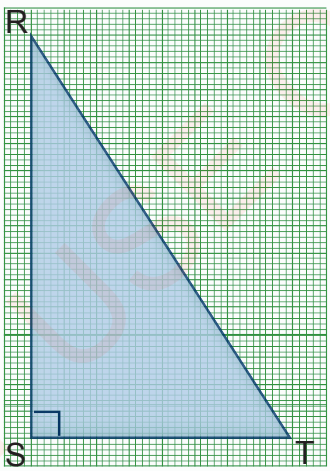


Calculation of lengths of the hypotenuse, height, and base of the right-angled triangle by using the Pythagoras' theorem

Two sides of the right-angled triangle can be used to calculate the third side by using the Pythagoras' theorem. The following examples illustrate the use of the Pythagoras' theorem in determining the length of one side of the right-angled triangle when the lengths of the two other sides are given.

Calculation of length of the hypotenuse

When the lengths of the base and height of the right-angled triangle are given, then you can find the length of the hypotenuse. If the length of the base is 'a', length of the height is 'b', and lenght of hypotenuse is 'c', then apply the Pythagoras' theorem $a^2 + b^2 = c^2$ to calculate the length of the hypotenus as follows:

Find the square root of each side as follows:

$\sqrt{a^2 + b^2} = \sqrt{c^2}$

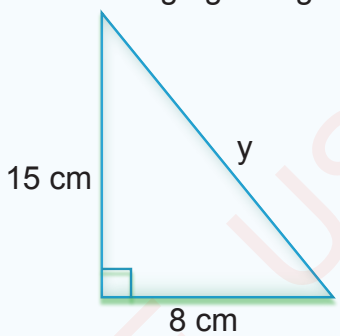
$\sqrt{a^2 + b^2} = c$

$c = \sqrt{a^2 + b^2}$.

Therefore, the length of the hypotenuse is equal to the square root of the sum of the squares of the lengths of base and height.

Example 1

Find the value of y in the following right-angled triangle:



Solution

Using the Pythagoras' theorem:

$a^2 + b^2 = c^2$

Base = a, Height = b and Hypotenuse = c.

From the figure:

c = y, a = 8 cm and b = 15 cm.

Thus,

$y^2 = (8\text{ cm})^2 + (15\text{ cm})^2$

$y^2 = 64\text{ cm}^2 + 225\text{ cm}^2$

$y^2 = 289\text{ cm}^2$.

Find the square root on both sides:

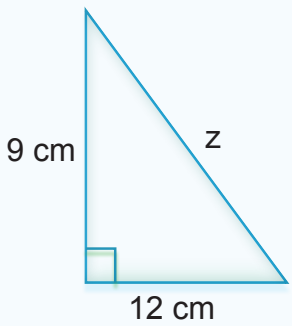
$\sqrt{y^2} = \sqrt{289\text{ cm}^2}$

$y = 17$ cm.

Therefore, the value of y is 17 cm.

Example 2

Find the value of z in the following right-angled triangle:



Solution

Using the Pythagoras' theorem:

$a^2 + b^2 = c^2$

Base = a, Height = b and Hypotenuse = c.

From the figure,

a = 12 cm, b = 9 cm and c = z.

Thus,

$z^2 = (12\text{ cm})^2 + (9\text{ cm})^2$

$z^2 = 144\text{ cm}^2 + 81\text{ cm}^2$

$z^2 = 225\text{ cm}^2$.

Find the square root on both sides

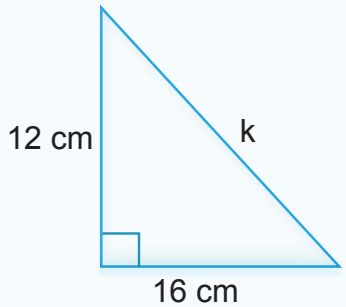
$\sqrt{z^2} = \sqrt{225\text{ cm}^2}$

$z = 15\text{ cm}$.

Therefore, the value of z is 15 cm.

Example 3

Find the value of k in the following right-angled triangle:



Solution

Using the Pythagoras' theorem:

$a^2 + b^2 = c^2$.

Base = a, Height = b and Hypotenuse = c.

From the figure,

a = 16 cm, b = 12 cm and c = k.

Thus,

$k^2 = (16\text{ cm})^2 + (12\text{ cm})^2$,

$= 256\text{ cm}^2 + 144\text{ cm}^2$

$= 400\text{ cm}^2$.

Find the square root on both sides:

$\sqrt{k^2} = \sqrt{400\text{ cm}^2}$

$k = 20$ cm.

Therefore, the value of k is 20 cm.

Example 4

Figure 11 shows a cyclist who intends to travel to village A from village B. She can choose to travel to village A from village B via village C in which she will cover a distance of 14 kilometres or she can take a direct route to village A. What distance will be covered if she takes a direct route to village A?

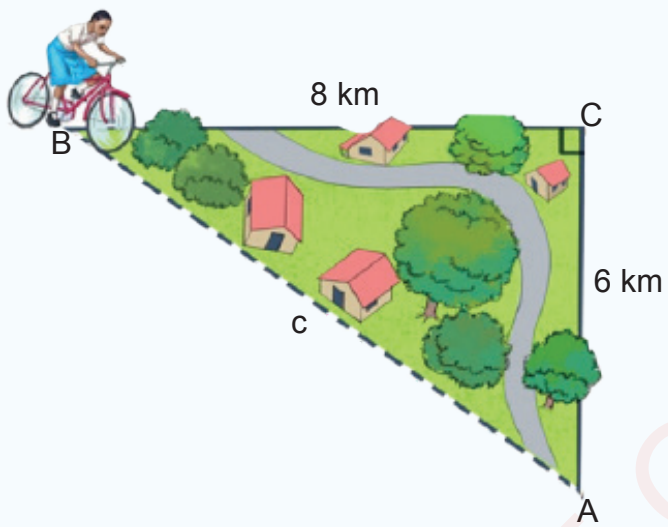


Figure 11: *Shortening the travelling distance by using the Pythagoras' theorem*

Solution

$\overline{BC}$ is the base with length a = 8 km.
$\overline{CA}$ is the height with length b = 6 km.
$\overline{BA}$ is the hypotenuse with length c km.
Find the value of c by using the Pythagoras' theorem:

$$\overline{BC}^2 + \overline{CA}^2 = \overline{BA}^2 \text{ or } c^2 = a^2 + b^2.$$

Thus,

$$c^2 = (8\text{ km})^2 + (6\text{ km})^2$$
$$= 64\text{ km}^2 + 36\text{ km}^2$$
$$c^2 = 100\text{ km}^2.$$

Find the square roots on both sides:

$$\sqrt{c^2} = \sqrt{100\text{ km}^2}$$
$$c = 10\text{ km}.$$

Therefore, the cyclist will ride 10 kilometres to village A from village B without passing through village C.

Exercise 3

Calculate the length of the hypotenuse for each right-angled triangle in the following questions:

1. Base = 15 m and height = 20 m.
2. Base = 10 cm and height = 24 cm.
3. Base = 20 cm and height = 21 cm.
4. Base = 7 m and height = 24 m.
5. Base = 18 cm and height = 24 cm.
6. Base = 5 m and height = 12 m.
7. Base = 16 cm and height = 30 cm.
8. Base = 21 m and height = 28 m.
9. Base = 3 m and height = 4 m.
10. Base = 6 cm and height = 8 cm.

Calculation of length of the height

The height of a right-angled triangle can be calculated if the lengths of the base and hypotenuse are known. If the length of base is 'a', length of height is 'b', and length of hypotenuse is 'c', then apply the Pythagoras' theorem $a^2 + b^2 = c^2$ to calculate the length of height 'b' as follows:

Subtract a^2 from both sides:

$$a^2 + b^2 - a^2 = c^2 - a^2$$
$$b^2 = c^2 - a^2.$$

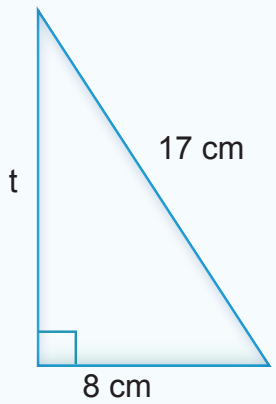
Find the square root on both sides of the equation:

$$\sqrt{b^2} = \sqrt{c^2 - a^2}$$
$$b = \sqrt{c^2 - a^2}.$$

Therefore, the length of height is equal to the square root of the difference between the squares of the lengths of hypotenuse and base.

Example 1

Find the value of t in the following right-angled triangle:



Solution

Using the Pythagoras' theorem:

$a^2 + b^2 = c^2$

Subtract a^2 from both sides:

$a^2 + b^2 - a^2 = c^2 - a^2$

$b^2 = c^2 - a^2$.

From the figure,

$a = 8$ cm, $b = t$ and $c = 17$ cm.

Thus,

$t^2 = (17 \text{ cm})^2 - (8 \text{ cm})^2$

$t^2 = 289 \text{ cm}^2 - 64 \text{ cm}^2$

$t^2 = 225 \text{ cm}^2$.

Find the square root on both sides:

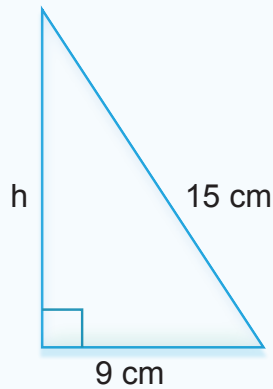
$\sqrt{t^2} = \sqrt{225 \text{ cm}^2}$

$t = 15$ cm.

Therefore, the value of t is 15 cm.

Example 2

Find the value of h in the following right-angled triangle:



Solution

By using the Pythagoras' theorem:

$a^2 + b^2 = c^2$.

Subtract a^2 from both sides:

$a^2 + b^2 - a^2 = c^2 - a^2$

$b^2 = c^2 - a^2$.

From the figure,

$a = 9$ cm, $b = h$ and $c = 15$ cm.

Thus,

$h^2 = (15 \text{ cm})^2 - (9 \text{ cm})^2$

$h^2 = 225 \text{ cm}^2 - 81 \text{ cm}^2$

$h^2 = 144 \text{ cm}^2$.

Find the square root on both sides:

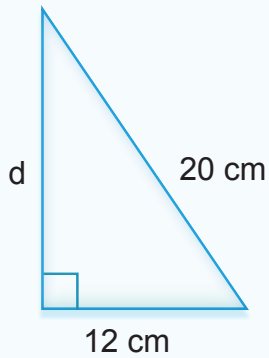
$\sqrt{h^2} = \sqrt{144 \text{ cm}^2}$

$h = 12$ cm.

Therefore, the value of h is 12 cm.

Example 3

Find the value of d in the following right-angled triangle:



Solution

Using the Pythagoras' theorem:

$a^2 + b^2 = c^2$.

Subtract a^2 from both sides:

$a^2 + b^2 - a^2 = c^2 - a^2$

$b^2 = c^2 - a^2$.

From the figure,

$a = 12$ cm, $b = d$ and $c = 20$ cm.

Thus,

$d^2 = (20 \text{ cm})^2 - (12 \text{ cm})^2$

$d^2 = 400 \text{ cm}^2 - 144 \text{ cm}^2$

$d^2 = 256 \text{ cm}^2$.

Find the square root on both sides:

$\sqrt{d^2} = \sqrt{256 \text{ cm}^2}$

$d = 16$ cm.

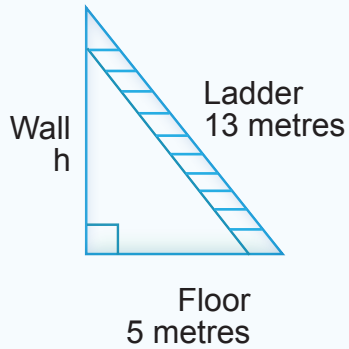
Therefore, the value of d is 16 cm.

Example 4

A ladder 13 metres long is placed against a vertical house wall. If the length between the foot of the ladder to the wall is 5 metres, find the height of the wall from the floor to the ladder.

Solution

Use the figure below:



The distance between the wall and the foot of the ladder is the base, which is a = 5 m.

The length of the ladder is the hypotenuse, which is c = 13 m.
The length of the height is b = h.

Using the Pythagoras' theorem:

$a^2 + b^2 = c^2$.

From the figure,

a = 5 m, b = h and c = 13 m

Thus,

$h^2 = (13\ m)^2 - (5\ m)^2$

$h^2 = 169\ m^2 - 25\ m^2$

$h^2 = 144\ m^2$.

Find the square root on both sides:

$\sqrt{h^2} = \sqrt{144\ m^2}$

h = 12 m.

Therefore, the height of the wall is 12 metres.

Exercise 4

Calculate the length of the height for each right-angled triangle in the following questions:

1. Base = 15 cm and hypotenuse = 25 cm.
2. Base = 8 m and hypotenuse = 17 m.
3. Base = 20 m and hypotenuse = 29 m.
4. Base = 7 cm and hypotenuse = 25 cm.
5. Base = 18 m and hypotenuse = 30 m.
6. Base = 16 cm and hypotenuse = 34 cm.
7. Base = 16 m and hypotenuse = 20 m.
8. Base = 21 cm and hypotenuse = 35 cm.
9. Base = 5 m and hypotenuse = 13 m.
10. Base = 8 cm and hypotenuse = 10 cm.

Calculation of length of the base

The length of the base of the right-angled triangle can be found if the lengths of height and hypotenuse are known. If the length of the base is 'a', length of the height is 'b', and length of hypotenuse is 'c', apply the Pythagoras' theorem $a^2 + b^2 = c^2$ to calculate the length of base 'a' as follows:

Subtract b^2 from both sides:

$$a^2 + b^2 - b^2 = c^2 - b^2$$
$$a^2 = c^2 - b^2.$$

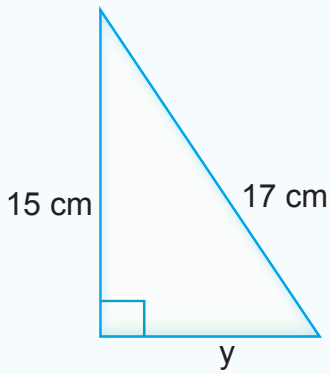
Find the square root on both sides:

$$\sqrt{a^2} = \sqrt{c^2 - b^2}$$
$$a = \sqrt{c^2 - b^2}.$$

Therefore, the length of the base is equal to the square root of the difference between the squares of the length of hypotenuse and height.

Example 1

Find the value of y in the following right-angled triangle:



Solution

Using the Pythagoras' theorem:

$a^2 + b^2 = c^2$.

Subtract b^2 from both sides:

$a^2 + b^2 - b^2 = c^2 - b^2$

$a^2 = c^2 - b^2$.

From the figure,

a = y, c = 17 cm and b = 15 cm

Thus,

$y^2 = (17\text{ cm})^2 - (15\text{ cm})^2$

$y^2 = 289\text{ cm}^2 - 225\text{ cm}^2$

$y^2 = 64\text{ cm}^2$.

Find the square root on both sides:

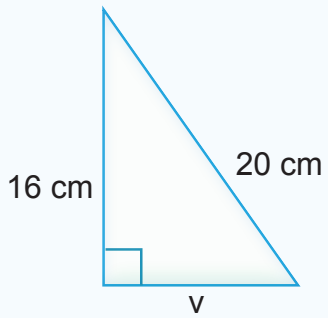
$\sqrt{y^2} = \sqrt{64\text{ cm}^2}$

$y = 8$ cm.

Therefore, the value of y is 8 cm.

Example 2

Find the value of v in the following right-angled triangle:



Solution

Using the Pythagoras' theorem:

$a^2 + b^2 = c^2$.

Subtract b^2 from both sides of the equation:

$a^2 + b^2 - b^2 = c^2 - b^2$

$a^2 = c^2 - b^2$.

From the figure,

$c = 20$ cm, $b = 16$ cm and $a = v$

Thus,

$v^2 = (20 \text{ cm})^2 - (16 \text{ cm})^2$

$v^2 = 400 \text{ cm}^2 - 256 \text{ cm}^2$

$v^2 = 144 \text{ cm}^2$.

Find the square root on both sides:

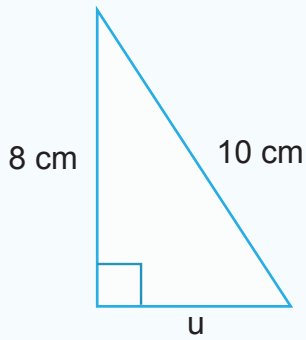
$\sqrt{v^2} = \sqrt{144 \text{ cm}^2}$

$v = 12$ cm.

Therefore, the value of v is 12 cm.

Example 3

Find the value of u in the following right-angled triangle:



Solution

Using the Pythagoras' theorem:

$a^2 + b^2 = c^2$.

Subtract b^2 from both sides:

$a^2 + b^2 - b^2 = c^2 - b^2$

$a^2 = c^2 - b^2$.

From the figure,

$c = 10$ cm, $b = 8$ cm and $a = u$.

Thus,

$u^2 = (10 \text{ cm})^2 - (8 \text{ cm})^2$

$u^2 = 100 \text{ cm}^2 - 64 \text{ cm}^2$

$u^2 = 36 \text{ cm}^2$.

Find the square root on both sides:

$\sqrt{u^2} = \sqrt{36 \text{ cm}^2}$

$u = 6$ cm.

Therefore, the value of u is 6 cm.

Exercise 5

Calculate the length of the base for each right-angled triangle with the following measurements:

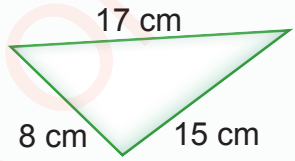
1. Height = 20 cm and hypotenuse = 25 cm
2. Height = 24 m and hypotenuse = 25 m
3. Height = 21 m and hypotenuse = 29 m
4. Height = 40 cm and hypotenuse = 41 cm
5. Height = 24 cm and hypotenuse = 30 cm
6. Height = 30 cm and hypotenuse = 34 cm
7. Height = 12 cm and hypotenuse = 20 cm
8. Height = 28 cm and hypotenuse = 35 cm
9. Height = 15 cm and hypotenuse = 17 cm
10. Height = 12 m and hypotenuse = 13 m

Revision Exercise

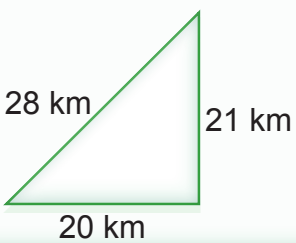
Answer the following questions:

1. Consider the triangles (i) to (x):
   (a) Identify the right-angled triangles in figures (i) - (x). Show how you obtained the answer.
   (b) Write the length of the hypotenuse of each right-angled triangle you have identified.

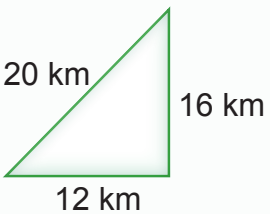
(i)



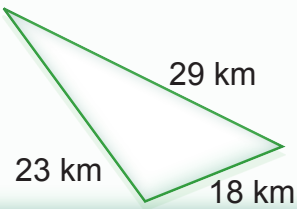
(ii)



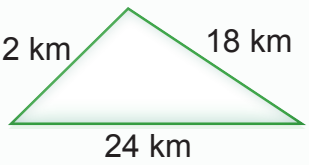
(iii)



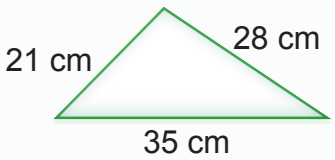
(iv)



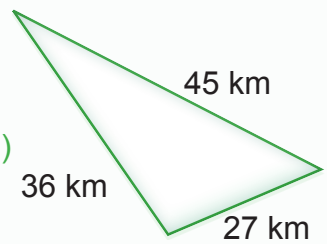
(v)



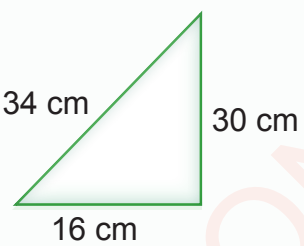
(viii)



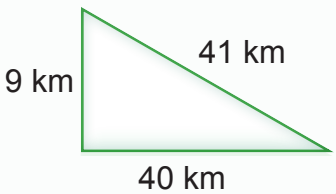
(vi)



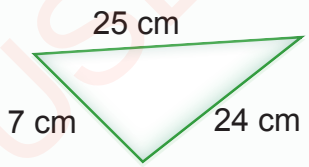
(ix)



(vii)



(x)



2. Find the length of the hypotenuse of each of the right-angled triangles with the following measurements:

(a) Base = 15 cm and height = 8 cm

(b) Base = 24 cm and height = 10 cm

(c) Base = 24 cm and height = 18 cm

(d) Base = 16 cm and height = 30 cm

3. Find the length of the height of the right-angled triangles with the following measurements:

(a) Base = 21 cm and hypotenuse = 35 cm

(b) Base = 24 cm and hypotenuse = 25 cm

(c) Base = 20 cm and hypotenuse = 29 cm

(d) Base = 12 cm and hypotenuse = 20 cm

4. Find the length of the base of the right-angled triangles with the following measurements:
   (a) Height = 24 cm and hypotenuse = 30 cm
   (b) Height = 16 cm and hypotenuse = 34 cm
   (c) Height = 8 cm and hypotenuse = 10 cm
   (d) Height = 7 cm and hypotenuse = 25 cm

5. A ladder leans against a vertical wall 6 metres apart. If the length of the ladder is 10 metres, find the height of the wall.

6. A football pitch has a length of 120 metres and width of 90 metres. A teacher assigned pupils to run by crossing the pitch diagonally. If they ran across and back, how many kilometres did the pupils run?

7. What is the distance from the top of a wall to the floor if a ladder of 15 metres long leans 12 metres away from the wall?

8. The area of a square is 100 square centimetres. Find the length of its diagonal.

9. Find the height of a rectangle if its base is 15 centimetres and its diagonal is 25 centimetres.

10. The distance of a mountain peak from a point at sea level is 5 kilometres. If the height of the mountain is 4 kilometres, find the width of that mountain at sea level.

11. Jane drives a car 28 kilometres south, and then 21 kilometres east. How far is Jane from her starting point?

12. Copy and fill the chart below if a is the base, b is the height, and c is the hypotenuse.

a	b	c
10	24	
15		39
	48	52

13. The ratio of the lengths of the base and height of a television is 3:4. If the length of its hypotenuse is 50 centimetres, find the length of the base and height of the television.

14. A company produced a right-angled triangle billboard for advertising educational facilities with a base of 24 centimetres and hypotenuse of 30 centimetres. Find the area of the billboard.

15. A canoeist at point A was crossing a river to point B. After crossing the river and due to strong winds, he discovered that he was at point C, 250 metres from A and 200 metres from B. If A, B and C fromed a right-angled triangle with the right angle at B, find the width of the river in centimetres.

Summary

1. The Pythagoras' theorem shows that the right-angled triangle is formed by three sides which are the hypotenuse, height, and base with the relation $a^2 + b^2 = c^2$; where 'a' is the length of base, 'b' is the length of height and 'c' is the length of hypotenuse.
2. Any triangle whose sides contradict with this theorem is not a right-angled triangle. If you have the right-angled triangle with the base of length 'a', height of length 'b', and hypotenuse of length 'c', using the Pythagoras' theorem, $c^2 = a^2 + b^2$, you will get $a^2 = c^2 - b^2$ and $b^2 = c^2 - a^2$.

Chapter Twelve

Business arithmetic

Introduction

In Standard Six, you learnt how to solve word problems which involve profit and loss. In this chapter, you will learn how to analyse sources of profit and loss, plan for income and expenditure, and design a project on income and expenditure. The competencies gained will help you to analyse the sources of profit and loss in sales and purchases. Also, you will be able to plan for proper expenditures on the available incomes, and finally design a project on sales and purchases using proper business practices.

Exercise 1: Revision

Answer the following questions:

1. A businessperson bought a crate of tomatoes for 55 000 shillings and sold it for 48 950 shillings.
   (a) How much loss did the businessperson make?
   (b) What percentage was the loss?

2. A profit of 10% was gained from selling sweets. If the profit was 3 500 shillings, what was the purchasing price of the sweets?

3. If a pair of shoes was sold at 16 000 shillings, the loss was 36%. What was the purchasing price of the shoes?

4. Saida bought 200 eggs. Assume that she bought every 10 eggs for 4 000 shillings and sold every 8 eggs for 4 000 shillings.
   (a) How much profit did she make?
   (b) What percentage was the profit?

5. A refrigerator was bought for 256 550 shillings and sold for 296 500 shillings.
   (a) How much profit was gained?
   (b) What percentage was the profit?

6. A blanket was bought for 64 000 shillings after a discount of 15%. What was its original price?

7. Jolie bought a goat for 72 200 shillings and slaughtered it. The goat produced 19 kilograms of meat. If she sold one kilogram of meat at 5 000 shillings,
   (a) did she make profit or loss?
   (b) what percentage was the profit or loss?

8. A certain secondary school raised 100 broilers. The price of each chick was 1 500 shillings. The broilers were kept for 90 days and used 15 bags of feed. Each bag costs 12 000 shillings. During all that time, they used chicken drugs which cost 35 000 shillings. If each chicken was sold at 7 000 shillings,
   (a) what profit or loss did the school make?
   (b) what percentage was the profit or loss?

9. A wooden bed was bought for 180 000 shillings. Find the selling price if it was sold at:
   (a) 20% profit.
   (b) 25% loss.

10. The price of one tin of maize is 15 000 shillings. If the price decreases by 5%, what will be the new price?

11. A loss of 16% was made from the sales of tomatoes. This loss was equivalent to 40 500 shillings. What were the actual sales of tomatoes?

12. Winifrida bought a sheep for 48 500 shillings and sold it for 42 750 shillings.
(a) What loss did she make?
(b) What percentage of the purchasing price was the loss?

13. Petro bought a vehicle for business. After three years he sold it at a profit of 12%. Find the selling price of the vehicle if he bought it for 24 000 000 shillings.

14. The price of a mobile phone is 1 250 000 shillings. If the price is discounted by $7\frac{1}{2}$%, what will the new price be?

15. Theodora bought a car for 58 540 000 shillings and sold it for 60 212 000 shillings. Alexander bought a motorbike for 2 850 000 shillings and sold it for 3 150 000 shillings.
(a) Find the percentage of profit for each person.
(b) Who made more profit? How much in percentage did the higher profit exceed the other?

16. Husna bought a small refrigerator for 265 000 shillings and sold it to Matilda at a loss of 8%. How much did Matilda pay for the refrigerator?

17. The commodities were bought for 400 000 shillings and sold for 500 000 shillings.
(a) Find the percentage of the profit made.
(b) Find the selling price at 16% profit.

18. On 24.01.2020, Grace spent her salary which was 485 000 shillings to buy the following items: 12 kg of meat @ sh 9 000, 15 kg of rice @ sh 2 500, 6 kg of groundnuts @ sh 3 000, 8 mangoes @ sh 1 500, 4 litres of milk @ sh 2 600, 14 bottles of soda @ sh 700, 3 kg of wheat flour @ sh 1 800, 5 kg of sugar @ sh 2 000, 4 cans of butter @ sh 4 500, and 20 litres of cooking oil @ sh 4 200.
(a) Prepare a bill for this information.
(b) Find the amount of money which Grace did not spend.

19. On 28.12.2019, Aloyce went to the shop with 355 000 shillings to buy the following items: 4 bundles of tomatoes @ sh 2 500, 3 bundles of onions @ sh 1 000, 5 bundles of carrots @ sh 2 000, 4 packets of yoghurt @ sh 1 500, 2 bags of maize flour @ sh 8 500, 5 kg of beans @ sh 2 800, 4 chicken @ sh 20 000, 10 eggs @ sh 400, and 2 brooms @ sh 3 400.

(a) Prepare a bill for this information.

(b) Find the amount of money which Aloyce did not spend.

20. On 28.01.2020, a treasurer collected a total of 36 850 000 shillings for Alphonce's wedding ceremony. The budget for the ceremony was as follows: 360 plates of food @ sh 12 000, reception hall for sh 2 400 000, 50 crates of soda @ sh 11 800, 4 cars for transport @ sh 170 000, a cake for sh 450 000, a master of ceremony for sh 750 000, 500 contribution cards @ sh 1 000, decorations for sh 300 000, 180 boxes of juice @ sh 2 500, 400 invitation cards @ sh 1 500, 30 cartons of water @ sh 3600, and 20% of the collection for wedding present.

(a) Prepare a bill for this information.

(b) Find the amount of money which was saved after the ceremony.

Activity 1: Analysing sources of profit and loss in business

Study and collect information on a business which generates profit and another one which makes losses. Answer the following questions:

(a) What are the sources of profit in business?

(b) What are the sources of loss in business?

(c) Give a piece of advice for the business that generates losses.

Sources of profit and loss

There are many sources of profit or loss on sales and purchase. One way of recognising the sources of profit or loss is to observe businesses that make profit or loss. The following are some of the reasons that can cause profit or loss on sales and purchases:

Infrastructure: If the infrastructure is bad, the cost of transportation will be high, the price of commodities will rise, and this in turn will reduce the number of customers. If it is good, transportation cost of commodities will be low. Thus, the price of commodities will be low and attract more sales.

Demands: When starting a business, it is important to find out the highly demanded commodities by the intended customers. This will increase the amount of sales and generate more profit. On the other hand, commodities with low demand will reduce the amount of sales and can be a source of loss.

Quality of commodities: A commodity of good quality attracts more customers and saves advertisement cost. Also, a quality commodity increases the amount of sales and generates more profit. On the contrary, low quality commodity does not attract customers. Thus, it will be a source of loss in the business.

Workers involved in business: Sometimes profit or loss in business depends on the qualification of workers involved in the business. Skilled, disciplined and trustful workers make business grow and generate profit. On the contrary, workers without these qualifications cause loss in the business.

Record keeping: Profit in a business can increase by keeping the records of income and expenditure properly. A businessperson who fails to keep records of income and expenditure may cause loss in the business. Improper record keeping slows down business and may lead into a total business collapse.

Activity 2: Importance of keeping records of income and expenditure

Instructions

Follow all the instructions that will be provided by your teacher to do this activity with your fellow pupils.

Income and expenditure

Income is the total amount of money obtained in a business in a specified duration of time. Sources of income in a business include:

(a) Producing and selling commodities;
(b) Purchasing and selling commodities;
(c) Providing services (example, taxi services);
(d) Receiving gifts from friends or families;
(e) Securing a loan; and
(f) Inheritance of properties.

Likewise, expenditure refers to payments made by the business in a specified period of time.

Importance of keeping records of income and expenditure

When a person or group of people decide to run a business, it is important for them to control the flow of money and commodities in the business. This is done by using books designed for keeping records of the business. Many businesspersons do not keep records of their business due to several reasons. One of the reasons is inability to understand proper ways of keeping records of the flow of money in the business. The human brain has a limit in storing records, thus, a person can easily loose important information. Consequently, a businessperson can forget the debtors, creditors, amount debited or credited. Therefore, keeping records is very important and it improves business.

The following are advantages of keeping records in business:

(a) It helps to know the amount of money received, and plan proper expenditure;
(b) It enables one to compute profit or loss and to know the capital in one's business;
(c) It enables people to keep information on expenses and debts. This will help to identify the debtors and the amount of money they owe. It also helps to know the debts one owes in the business;
(d) It prevents misuse of funds and mistrust among business owners; and
(e) It helps in monitoring and controlling commodity stores, so as to easily identify any new commodities to be ordered.

Managing income and expenditure in a business

Good keeping of income and expenditure records is done professionally using ledgers or properly tabulated exercise books.

The following are advantages of good record keeping of income and expenditure in a business:

(a) Helps to know the profit or loss made in the business during a specific period of time;
(b) Helps to know the available business resources and dividends;
(c) Helps to know the creditors and debtors in the business;
(d) Helps businesspersons in making decisions on whether to increase or decrease the capital in the business;
(e) Helps to know the amount of tax to be paid;
(f) Helps financial institutions to make decisions on loan applications, and
(g) Helps to know the rate at which the business is growing or depreciating.

Managing cash books

Human beings are engaged in different activities for the purpose of generating income. The income can be in form of cash or asset. Thus, it is important to have proper records of income and expenditure of the available resources. Proper record keeping of income and expenditure helps to know whether profit or loss is being generated. Since the human brain has a limit in storing information, keeping records is very important in business. Governments, institutions, farmers, businesspersons, and families keep records of their income and expenditure. The process of keeping records in a cash book involves recording all business transactions in terms of cash and assets.

Cash books

The books which are used to record transactions are called ledgers. An exercise book can be used to record transactions if it is tabulated in a ledger form. The parts of a ledger are shown in Table 1.

Dr Cr

Date	Particulars	Folio	Amount		Date	Particulars	Folio	Amount	
			sh.	ct.				sh.	ct.

Table 1: *Parts of a ledger*

Business arithmetic has two main sides, namely, DEBIT (Dr) and CREDIT (Cr). DEBIT (Dr) is the side that represents income in the business. CREDIT (Cr) is the side that represents expenditure in the business. Both sides have columns for Date, Particulars, Folio, and Amount (shillings [sh] and cents [ct]). The Particulars column represents types of transactions, the Folio column represents the number of the entry, and the Amount column represents the money involved. The capital and all income transactions are recorded on the debit side. The expenditures and other expenses are recorded on the credit side.

FOLIO represents numbers assigned to similar particulars (items). For example, all sales are assigned the same number.

In order to show the balance carried down (balance c/d) and balance brought down (balance b/d), the difference in the total amounts in the DEBIT and CREDIT sides is computed. If the amount in the DEBIT side is larger, then the difference is recorded and added on the CREDIT side. The total amount on the two sides of the ledger must be equal. The difference shows the balance at the end of a specific period of doing business, for example, a month or year. This balance is considered as the balance available for business in the next period of business. If the amount in the CREDIT side is larger, then the difference is recorded and added on the DEBIT side. If the amounts in the DEBIT and CREDIT sides are equal, then the business will have neither balance carried down nor balance brought down.

Example 1

Angelina's business transactions for September 2018 were as follows :
September 1, 2018 had a capital of 3 450 000 shillings cash;
September 2, purchased goods for 280 500 shillings;
September 3, paid transport of goods for 21 500 shillings;
September 5, sold goods for 85 000 shillings;
September 8, sold goods for 122 000 shillings;
September 12, hired a car for 51 000 shillings;
September 14, purchased goods for 185 000 shillings;
September 16, sold goods for 295 000 shillings;
September 18, received 114 300 shillings from sales of goods;
September 20, sold goods for 116 000 shillings;
September 23, paid salary of 140 000 shillings to a shopkeeper;
September 24, paid rent of 250 000 shillings;
September 26, paid15 000 shillings for electricity bill;
September 28, purchased goods for 625 000 shillings;
September 29, purchased goods for 600 000 shillings.
(a) Record the transactions in a ledger using the double entry principle.
(b) Indicate the balance carried down and the balance brought down.

Solution

Angelina's Cash A/C (1)

Dr					Cr				
Date	Particulars	Folio	Amount		Date	Particulars	Folio	Amount	
			sh	ct				sh	ct
2018					2018				
Sept 1	Capital	2	3 450 000	00	Sept 2	Purchases	3	280 500	00
Sept 5	Sales	5	85 000	00	Sept 3	Transport	4	21 500	00
Sept 8	Sales	5	122 000	00	Sept 12	Transport	4	51 000	00
Sept 16	Sales	5	295 000	00	Sept 14	Purchases	3	185 000	00
Sept 18	Sales	5	114 300	00	Sept 23	Salary	6	140 000	00
Sept 20	Sales	5	116 000	00	Sept 24	Rent	7	250 000	00
					Sept 26	Electricity	8	15 000	00
					Sept 28	Purchases	3	625 000	00
					Sept 29	Purchases	3	600 000	00
					Sept 30	Balance c/d		2 014 300	00
			4 182 300	00				4 182 300	00
Oct 1	Balance b/d		2 014 300	00					

Example 2

Prepare a cash book for Mustapha's income and expenditure for July, 2019. Indicate balance brought down as of 1st August, 2019:
July 1, start up capital of 350 000 shillings cash;
July 2, purchased goods for 210 000 shillings;
July 4, sold goods for 112 400 shillings;
July 7, sold goods for 172 550 shillings;
July 10, purchased goods for 120 100 shillings;
July 10, paid 12 000 shillings for transport of goods;
July 14, purchased goods for 92 600 shillings;
July 16, sold assets for 275 700 shillings;
July 19, repaired shop building worthy 52 000 shillings;
July 21, bought goods for 183 150 shillings;
July 24, paid 130 000 shillings for salary;
July 25, paid 70 000 shillings for shop license;
July 26, sold goods for 196 900 shillings;
July 28, paid 10 000 shillings for electricity bill;
July 29, hired a car for 42 000 shillings;
July 29, received 223 850 shillings from sales of goods.

Solution

Mustapha's cash A/C (1)

Dr					Cr				
Date	Particulars	Folio	Amount		Date	Particulars	Folio	Amount	
			sh	ct				sh	ct
2019					2019				
July 1	Capital	2	350 000	00	July 2	Purchases	3	210 000	00
July 4	Sales	4	112 400	00	July 10	Purchases	3	120 100	00
July 7	Sales	4	172 550	00	July 10	Transport	5	12 000	00
July 16	Sales	4	275 700	00	July 14	Purchases	3	92 600	00
July 26	Sales	4	196 900	00	July 19	Maintenance	6	52 000	00
July 29	Sales	4	223 850	00	July 21	Purchases	3	183 150	00
					July 24	Salary	7	130 000	00
					July 25	License	8	70 000	00
					July 28	Electricity	9	10 000	00
					July 29	Transport	5	42 000	00
					July 31	Balance c/d		409 550	00
			1 331 400	00				1 331 400	00
August 1	Balance b/d		409 550	00					

Exercise 2

Answer the following questions:

1. Prepare a cash book for the following information:
   On December 4, 2017, Bakari had 270 000 shillings cash; December 5, bought beef stew for 12 500 shillings; December 9, bought tomatoes for 4 500 shillings; December 12, sold cabbages for 22 500 shillings; December 15, bought rice for 35 000 shillings; December 23, sold vegetables for 6 500 shillings; December 26, sold a goat for 81 000 shillings; December 27, paid 60 000 shillings for rent, and 9 450 shillings for water bill.

2. On October 3, 2016, a poultry farmer had 500 000 shillings cash; October 6, bought chicken feed for 60 000 shillings; October 13, used 5 200 shillings to buy veterinary drugs; October 19, bought a wire mesh for 122 500 shillings; October 23, bought broiler chicks for 300 000 shillings; October 25, bought chicken feed for 48 000 shillings; and October 26, sold chickens for 700 500 shillings.
   (a) Record these transactions in a ledger using the double entry principle.
   (b) What was the balance brought down?

3. On January 1, 2020, a businessperson started a vegetable business with a capital of 650 000 shillings in cash.
   January 2, bought vegetable seedlings for 12 500 shillings;
   January 3, purchased fertilisers for 17 500 shillings;
   January 4, paid 8 300 shillings cash for transport;
   January 5, paid 18 750 shillings cash for vegetable seedlings;
   January 7, paid 6 000 shillings cash for water bill;
   January 9, bought fertilisers for 21 400 shillings;
   January 11, bought pesticides for 44 500 shillings;
   January 14, paid 30 000 shillings for wages;
   January 17, paid 12 000 shillings for electricity bill;
   January 20, received 48 500 shillings for sales of vegetables;
   January 22, received 36 000 shillings for sales of vegetables;

January 25, bought fertilisers for 25 000 shillings;

January 28, sold vegetables for 42 000 shillings.

(a) Record these transactions in a ledger using the double entry principle.

(b) Show the balance carried down and balance brought down.

4. The income and expenditure of Nancy in November, 2016 were as follows:

Date	Particulars
November 1	Savings in cash 942 800 shillings
November 4	Bought goods for 548 750 shillings
November 7	Purchased goods for 110 300 shillings
November 9	Transported goods for 25 000 shillings
November 12	Sold goods for 302 000 shillings
November 13	Sold goods for 284 200 shillings
November 16	Bought goods for 74 500 shillings
November 19	Transported goods for 15 500 shillings
November 21	Paid 21 750 shillings cash for water bill
November 22	Bought goods for 192 500 shillings
November 24	Bought a lantern for 18 000 shillings
November 25	Paid 180 000 shillings cash for salary
November 27	Paid 200 000 shillings cash for rent
November 29	Paid 100 000 shillings cash for day workers

Record the above transactions for November, 2016 in a ledger. Show the balance carried down and balance brought down.

Activity 3: Project on sales and purchase

Instructions

1. Design a business that starts from 1st to 31st January this year.
2. Make transactions on sales and purchases.
3. Record all the transactions in a ledger.
4. Show the balance carried down and balance brought down.
5. Follow the instructions given by your teacher, to discuss with your fellow pupils.

Profits and costs of depositing and borrowing money

In business, you can deposit money or secure a loan from banks, micro financial institutions, SACCOS or individuals offering financial services. If you deposit money for a specified period of time, you will generate profit depending on the amount of money deposited. The profit generated is called interest. Profit is computed basing on the interest rate paid by the service provider. You can also secure a loan for business purposes. The loan will be repaid with an interest based on the agreed interest rate and fixed time period.

Interest (I), interest rate (R), time (T) and principal (P) in bank deposits

In this section, you will learn how to solve word problems on profit. Besides, you will learn about interest (I), interest rate (R), time (T), and principal (P). The amount of money deposited in a bank for business is called principal. The principal generates a profit which is called interest, when deposited in bank for a fixed period of time. The annual interest rate is given in percentage. A person who deposits money in the bank expects to earn interest based on the amount of money deposited. The interest (I) earned depends on the principal (P), interest rate (R), and the fixed time (T).

Computations of interest, interest rate, time, and principal

Interest generated from bank deposits is computed by using the following formula:

$$\text{Interest (I)} = \frac{\text{Principal (P)} \times \text{Rate (R)} \times \text{Time (T)}}{100}$$

Or, in short form: $I = \frac{P\,R\,T}{100}$.

By using this formula, you can get the formulae for other variables as follows:

Interest (I)	**Interest rate (R)**
$I = \frac{P\,R\,T}{100}$	Using $I = \frac{P\,R\,T}{100}$, Multiply by 100 on both sides $100 \times I = \frac{P\,R\,T}{100} \times 100$ $100I = PRT$ Divide both sides by PT $\frac{100I}{PT} = \frac{PRT}{PT}$ $R = \frac{100I}{PT}$
Principal (P)	**Time (T)**
Using $I = \frac{P\,R\,T}{100}$, Multiply both sides by 100 $100 \times I = \frac{P\,R\,T}{100} \times 100$ $100I = PRT$ Divide both sides by RT $\frac{100I}{RT} = \frac{PRT}{RT}$ $P = \frac{100I}{RT}$	Using $I = \frac{PRT}{100}$, Multiply both sides by 100 $100 \times I = \frac{PRT}{100} \times 100$ $100I = PRT$ Divide both sides by PR $\frac{100I}{PR} = \frac{PRT}{PR}$ $T = \frac{100I}{PR}$

Example 1

Find the interest earned after depositing 40 000 shillings in a bank at an interest rate of $10\frac{1}{2}$ % per year for 4 years.

Solution

Given Principal, P = sh 40 000

Interest rate, R = $10\frac{1}{2}$ % or $\frac{21}{2}$%

Time, T = 4 years

By using the formula for computing interest, $I = \frac{PRT}{100}$

Substitute in the formula all values of variables as follows:

$$I = \frac{\text{sh } 40\,000 \times 21 \times 4}{2 \times 100}$$
$$= \text{sh } 400 \times 21 \times 2$$
$$= \text{sh } 16\,800$$

Therefore, the interest earned was 16 800 shillings.

Example 2

Amina deposited 200 000 shillings in a bank. If the bank pays an annual interest rate of $7\frac{1}{2}\%$. How long did it take for her to earn an interest of 45 000 shillings?

Solution

Given P = sh 200 000, I = sh 45 000, $R = 7\frac{1}{2}\% = \frac{15}{2}\%$, T = ?

By using the formula for computing time: $T = \frac{100I}{PR}$

Substitute in the formula all values of variables as follows:

$$T = \frac{100 \times \text{sh } 45\,000 \times 2}{\text{sh } 200\,000 \times 15}$$
$$= \frac{45}{15} = 3$$

T = 3 years.

Therefore, Amina received an interest of 45 000 shillings after 3 years.

Example 3

Sekela deposited money in a bank that pays an annual interest rate of 10 %. If she received an interest of 21 600 shillings after 3 years, how much money did she deposit?

Solution

Given I = sh 21 600, T = 3 years, R = 10%, P =?

By using the formula for computing principal, $P = \frac{100I}{RT}$

Substitute the values of variables in the formula as follows:

$$P = \frac{100 \times \text{sh } 21\,600}{10 \times 3}$$

$$= \text{sh } 72\,000$$

Therefore, Sekela deposited 72 000 shillings.

Example 4

An amount of 360 000 shillings was deposited in a bank. An interest of 129 600 shillings was generated after 4 years. What is the interest rate paid by the bank per year?

Solution

Given P = sh 360 000, I = sh 129 600, T = 4 years, R = ?

Using the formula for computing interest rates, $R = \frac{100I}{PT}$

Substitute the values of variables in the formula as follows:

$$R = \frac{100 \times \text{sh } 129\,600}{\text{sh } 360\,000 \times 4}$$

$$= 9\%$$

Therefore, the bank pays an annual interest rate of 9%.

Exercise 3

Answer the following questions:

1. Fill in the blanks in the following table:

	Principal, P (sh)	Interest rate, R (%)	Time, T (years)	Interest, I (sh)
(a)	360 000	$9\frac{1}{2}$	2	
(b)		10	5	65 500
(c)	4 200 000		6	2 016 000
(d)	850 000	12		51 000
(e)	680 000	$7\frac{1}{2}$	4	

2. A businessperson deposited 550 000 shillings in a bank for 2 years. What is the interest gained if the bank pays an interest rate of 10% per year?

3. Anton deposited his money in a bank that pays an interest rate of 12% per year. If it took $3\frac{1}{2}$ years to get an interest of 420 000 shillings, how much money did he deposit?

4. An amount of 148 000 shillings was deposited in a bank which pays an interest rate of $8\frac{1}{2}$% per year. Compute the interest generated after 9 months.

5. Mwajuma deposited 364 000 shillings in a bank which pays an interest rate of $5\frac{1}{2}$% per year. What interest did she get after 6 months?

6. Catherine deposited 1 200 000 shillings in a bank. She received an interest of 50 000 shillings after 4 months. Find the interest rate paid by the bank per year.

7. Abdallah deposited 20% of his 500 000 shillings savings in a bank that pays an interest rate of 12% per year. If he received an interest of 36 000 shillings, for how long did he deposit his money?

8. Asha deposited 320 000 shillings in a bank. After one year, she received an interest of 41 600 shillings. Find the interest rate paid by the bank per year?

9. Joyce deposited her money in a bank. An interest of 160 000 shillings was generated after 8 months. How much money did she deposit if the bank pays an interest rate of 12% per year?

10. Halima deposited 240 000 shillings in a bank which pays an interest rate of $8\frac{1}{2}\%$ per year. How long did it take for her to receive an interest of 10 200 shillings?

11. Mabula deposited 450 000 shillings in a bank. After 5 years, he received an interest of 258 750 shillings. Find the interest rate paid by the bank per year.

12. An interest of 103 800 shillings was gained from a bank deposit of 432 500 shillings. Find the interest rate paid by the bank if the money was fixed for a period of 2 years and eight months.

13. Debora deposited her money in a bank that pays an interest rate of $6\frac{1}{2}\%$ per year. If she received an interest of 884 000 shillings after 34 months, how much money did she deposit?

14. Apolinary deposited 1 600 000 shillings in a bank that pays an interest rate of 6% per year. How long did he deposit the money if he received an interest of 336 000 shillings?

15. An amount of 450 000 shillings was deposited in a bank and generated an interest of 522 000 shillings after 5 years. Find the interest rate paid by the bank per year.

16. Jenifer deposited 180 000 shillings in a bank which pays an interest rate of 8% per year. What interest did she receive after 5 years?

17. Bundala deposited 2 400 000 shillings in a bank which pays an interest rate of 14% per year. If he received an interest of 1 512 000 shillings, for how long did he deposit the money?

18. Mathias secured a loan of 600 000 shillings in a bank which charges an interest rate of 12% per year. What interest will he pay after 3 years and 9 months?

19. Juma deposited 1 200 000 shillings in a bank which pays an interest rate of $12\frac{1}{2}$ % per year. How much money will he have in his account after 4 years?

20. Alex secured a loan of 9 000 000 shillings in a bank which charges an interest rate of 18% per year. He repaid the loan after 2 years and 6 months. What total amount of money did he pay to the bank?

Revision exercise

Answer the following questions:

1. Find the missing values of the letters in the following questions:

(a) P = sh 248 000	R = 12%	T = 4 years	I = ?
(b) P = ?	R = 8%	T = 2 years	I = sh 40 000
(c) P = sh 120 000	R = 10%	T = ?	I = sh 24 000
(d) P = sh 40 925 000	R = ?	T = 3 years	I = sh 24 555 000

2. Ashura deposited 16 800 000 shillings in a bank which pays an interest rate of 10% per year. If she received an interest of 4 200 000 shillings, what was the fixed time period of the deposit?

3. How do you compute the balance carried down and the balance brought down after recording transactions in a ledger?

4. Explain the difference between DEBIT (Dr) and CREDIT (Cr) as used in business arithmetic.

5. Getrude deposited 850 000 shillings in a bank. She received an interest of 382 500 shillings after 4 years. Find the interest rate paid by the bank per year.

6. Imani borrowed 3 000 000 shillings from a bank which charges an interest rate of 15% per year. What amount of interest did he pay after five years and eight months?

7. Explain why quality of commodities is a source of profit generation in a business?

8. A social development group borrowed 3 500 000 shillings from a bank which charges an interest rate of 16% per year. How long will it take for the group to repay an interest of 1 120 000 shillings?

9. A certain SACCOS borrowed 45 000 000 shillings from a bank to lend to its members. If the bank charges an interest rate of 12% per year, what is the interest to be paid after 6 years?

10. A certain local food vendor deposited 800 000 shillings in a bank. She received an interest of 336 000 shillings after 3 years. What was the interest rate paid by the bank per year?

11. Richard deposited his money in a bank which pays an interest rate of 10% per year. If he received an interest of 6 969 000 shillings after 3 years, how much money did he deposit?

12. A sisal company borrowed 250 000 000 shillings from a bank which charges an interest rate of $12\frac{1}{2}\%$ per year. What interest will be paid by the company after 5 years?

13. Mention three advantages of good record keeping of income and expenditure.

14. Mention three benefits gained from good record keeping in business.

15. Elena secured a loan of 12 000 000 shillings from a bank which charges an interest rate of 12% per year. She paid an interest of 2 400 000 shillings after 2 years as part of the loan. What amount of money did she pay?

16. List four sources of income in business.

17. Record the following transactions in a ledger of Selina's business in December, 2015. Show the balance carried down and balance brought down.

Date	Particulars	Amount (sh)
December 01	Shop savings	347 000
05	Sold goods	272 000
07	Paid insurance	82 000
12	Bought furniture	146 000
15	Sold goods	120 500
19	Purchased goods	180 000
21	Purchased goods	100 000
23	Sold goods	118 700
24	Sold goods	75 000
25	Paid rent	164 000
26	Paid salary to Sonzo	67 000
27	Sold goods	125 500
28	Purchased goods	220 000

18. The income and expenditure of Zainabu in January 2013 were as follows:

Date	Particulars	Amount (sh)
1	Capital in cash	700 000
3	Purchased goods	360 000
5	Sold carvings	450 000
7	Bought carvings	245 000

11	Carvings sales	280 000
13	Bought a cupboard	45 500
16	Salary for a watchman	54 500
18	Purchased goods	235 500
19	Sales	150 450
21	Carriages	34 500
22	Rent	25 000
23	Sales	120 600
24	Purchased goods	225 000
25	Vendor's salary	100 000
26	Sales	92 800
27	Paid electricity and water bills	25 000

Prepare a ledger for these transactions, and show the balance carried down and balance brought down.

19. Record the following transactions for January, 2014 in a cash book. Show the balance carried down and balance brought down.

Date	Particulars	Amount (sh)
1	A livestock keeper's capital in cash	630 800
3	Bought veterinary drugs	205 000
4	Paid cash for cattle treatment	144 500
6	Milk sales	165 850
8	Repairing cattle shed	132 000
12	Salaries of attendants	220 000
14	Milk sales	140 700
19	Paid water and electricity bills	35 000
23	Manure sales	52 300
26	Calves sales	240 000
29	Milk sales	85 000

20. In November 1, 2019, Edward had a capital of 942 800 shillings. November 2, he purchased goods for 510 400 shillings and paid

transport for 24 500 shillings. November 4, sold goods for 128 350 shillings and paid 95 500 shillings cash for electricity installations. November 6, he made sales of 258 400 shillings. November 9, he made purchases of 245 200 shillings and paid 16 000 shillings cash for transport. November 12, he received 327 200 shillings from sales. November 15, he paid 21 000 shillings cash for electricity and sold assets for 186 200 shillings. November 19, he purchased goods for 600 750 shillings and paid 10 000 shillings cash for transport. November 22, he made sales of 321 400 shillings. November 26, he paid 184 000 shillings cash for attendants salaries and received sales of 94 700 shillings. Record Edward's transactions in a ledger and show the balance carried down and balance brought down.

Summary

1. When computing the interest rate, principal or time, use the interest formula, $I = \frac{PRT}{100}$.
2. In business, things which can influence your profit or loss include infrastructure, demand, and quality of commodities.
3. When doing business, make sure that cash books are recorded in a ledger to show income and expenditure.
4. Income and expenditure records help to know the amount of profit or loss generated in a business.

Chapter Thirteen

Statistics

Introduction

Statistics is a mathematical science that deals with collection, analysis, and interpretation of data. Graphical representations of statistical data are drawn as pictograms, bar graphs, pie charts, and line graphs. In the previous classes, you learnt how to draw pictorial representation, bar graphs, and pie charts. In this chapter, you will learn how to draw and interpret line graphs by using available information. Also, you will learn how to carry out projects using various data following given criteria. The competencies gained will help you to collect, analyse, and interpret data in different fields for instance; business, insurance, ecology (such as finding out the interaction between living things and the environment), health (such as the analysis of diseases), demography (such as human populations, birth, and death rates) and astronomy. Figure 1 shows the relationship between wildlife and the environment. For instance, information about the availability of food and water in a certain area in relation to the birth and death rates of animals can be presented easily using the knowledge of statistics. Therefore, statistics will help you to analyse and interpret data of those relationships in real life.

Figure 1: *Wildlife in relation to the environment*

Exercise 1: Revision

Answer the following questions:

1. Amos scored the following marks in Mathematics tests:
In Test one he scored 97 percent, in test two he scored 100 percent, and in test three he scored 85 percent. What is the average mark?

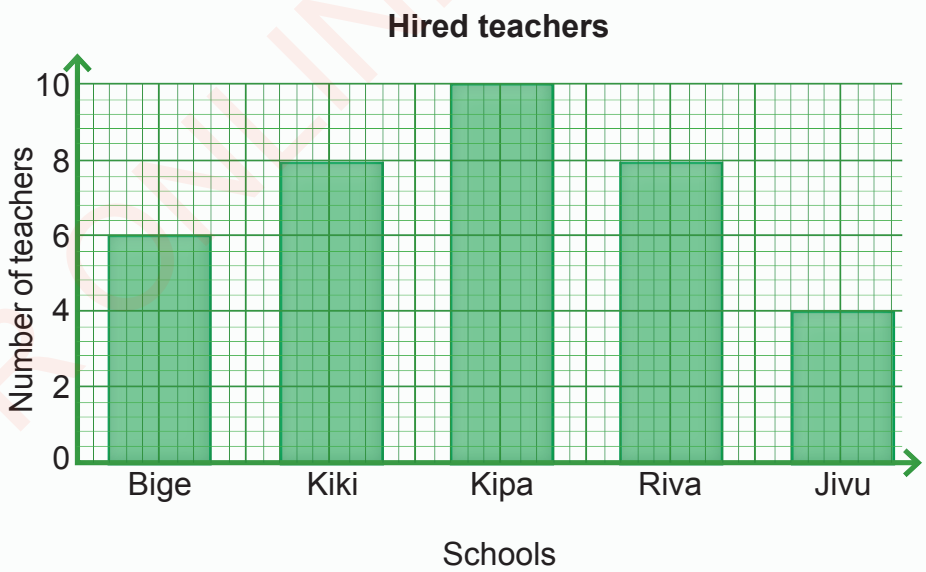
2. The average height of pupils in five classes was as follows:

Class	Three	Four	Five	Six	Seven
Height	125 cm	1.28 m	130 cm	1.32 m	135 cm

Find the average height of the pupils in all classes.

3. A motorbike carried passengers of different masses in different times as follows: 60 kg, 85 000 g, 0.065 ton, 50 kg, 70 000 g, 45 kg, and 0.045 ton. What is the average mass of the passengers carried by the motorbike?

4. The following graph shows the number of hired teachers in five primary schools.



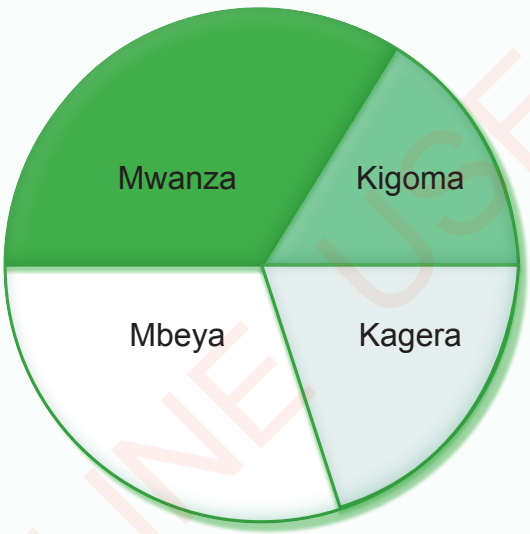
(a) Which school hired the largest number of teachers?

(b) Which school hired the least number of teachers?

(c) Find the difference between the number of teachers in the school that hired the largest number and that which hired the least number of teachers.

(d) Find the total number of the teachers hired in all five schools.

5. A bus company transported a total of 3 600 passengers from Mwanza, Kagera, Kigoma and Mbeya to Dar es Salaam region. Study the following pie chart and answer the questions that follow:



(a) What is the size of the angle for each region?

(b) Find the total number of passengers from each region.

(c) Which region transported the least number of passengers?

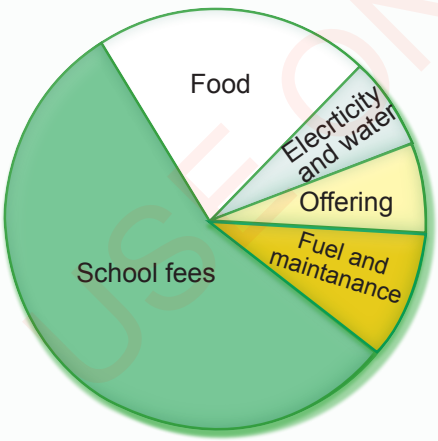
6. Maendeleo Ward has 25 male teachers, 20 female teachers and 15 non-teaching staff. Present this information in a pie chart.

7. A farmer's monthly expenditure is as shown in the following table:

Expenditure	Percentage (%)
Food	20
Rent	15
Fare	12
Communication	5
Savings	38
Emergency	5
Offering	5

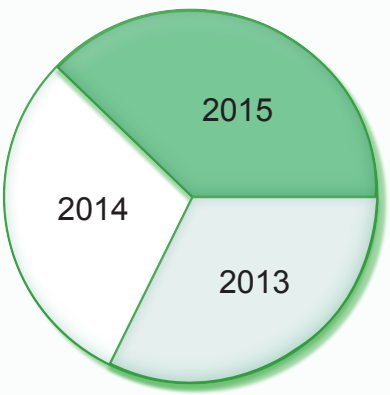
Draw a pie chart to represent this information.

8. A teacher at Mapinduzi Primary School earns 1 800 000 shillings as a monthly take home salary. The teacher's monthly expenditure is as shown in the following pie chart.



(a) What is the size of the angle for each expenditure?
(b) Find the amount of money for each expenditure.
(c) Calculate and list each expenditure in percentage.

9. The following pie chart shows 36 000 pupils who were enrolled in Form One from the year 2013 to 2015 in a certain region.
(a) Measure and write the size of the angle for each year.
(b) Find the number of pupils enrolled in Form One in each year.



(c) Mention the year which enrolled the largest number of pupils in Form One.

(d) Mention the year which enrolled the least number of pupils in Form One.

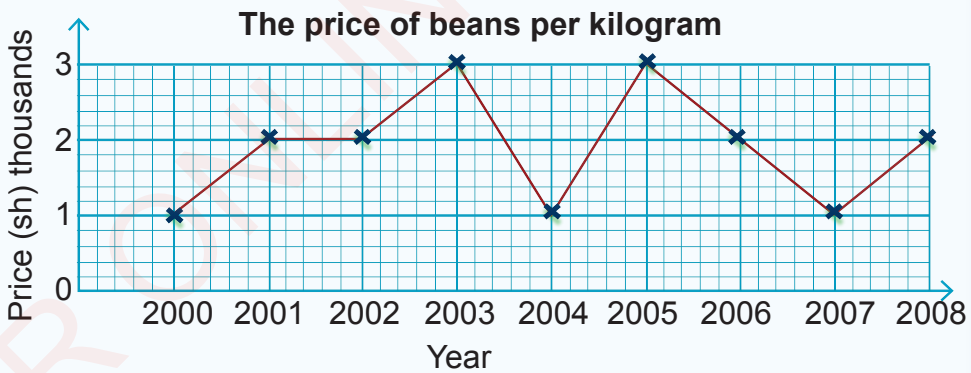
Line graphs

A line graph represents the relationship between two variables such as blood group and age, or number of events and time. A line graph uses lines to show the relationship between two variables in the x-axis and y-axis. The line graph normally includes the units of a physical quantity represented in the graph. For example, when the line graph represents mass in one axis, the unit of mass like ton, kilogram or gram is shown along that axis. Line graphs can be used in hospitals to show clinical results, in schools to display the pupil's marks and in business to show sales of different commodities.

Reading and interpreting line graphs

Example

Study the following graph, and then answer the questions that follow.



(a) Mention the scale of the graph in the x-axis and y-axis.

(b) What was the price of 1 kilogram in 2008?

(c) In which years was the price of beans 1 000 shillings?

(d) Mention the years in which the price of beans did not change.

(e) In the year 2003, a family bought 50 kilograms of beans. How much money did the family spend?

Answers

(a) The horizontal line (x-axis) represents sales of beans in a year.
Scale: One unit represents one year.
The vertical line (y-axis) represents the price of beans in shillings.
Scale: One unit represents one thousand shillings.

(b) To get the cost of one kilogram of beans, read the point where 2008 is located on the x-axis. Move upwards along the y-axis until you meet the line. Move leftwards to the y-axis, read the price of one kilogram of beans. The value of y-axis is sh $2 \times 1\,000 =$ sh 2 000. Therefore, the price of 1 kilogram of beans was 2 000 shillings.

(c) To get the years in which the cost of one kilogram of beans was 1 000 shillings, read the point where 1 000 shillings is located on the y-axis. Move rightwards until you meet the line, then move downwards to meet the x-axis and read the year. The years with this price were 2000, 2004 and 2007. Therefore, the years where the cost of one kilogram of beans was 1 000 shillings were 2000, 2004 and 2007.

(d) The points within the line graph which lie in the same line along the x-axis had the same price. Therefore, the two years which had the same price are 2001 and 2002.

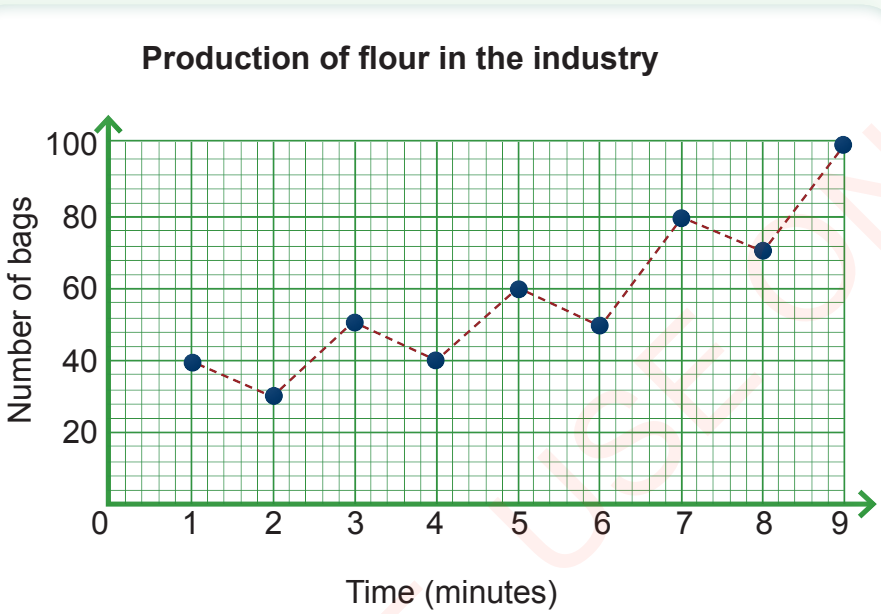
(e) In the year 2003, the price of beans was 3 000 shillings. Multiply the number of kilograms by the price of one kilogram of beans. Thus, 50 kg $\times$ sh 3000 per kg = sh 150 000.

Therefore, the family paid 150 000 shillings.

Exercise 2

Answer the following questions:

1. The following graph shows the number of bags of flour produced per minute by a flour mill.



(a) How many bags of flour were produced in the:

(i) First minute?

(ii) Sixth minute?

(iii) Eighth minute?

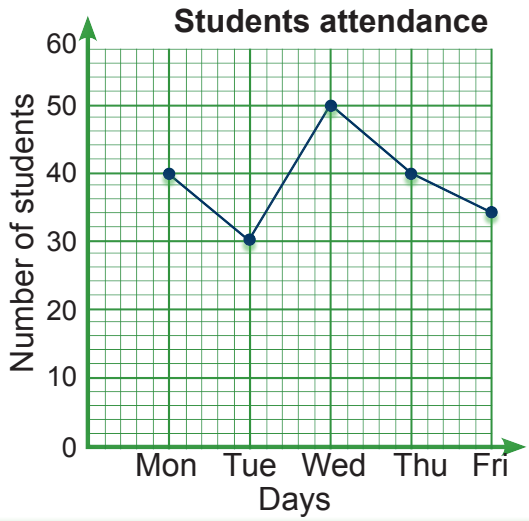
(b) In which minute did the flour mill produce:

(i) 30 bags of flour?

(ii) 60 bags of flour?

(iii) 80 bags of flour?

2. The following graph shows the attendance of Standard Six pupils per week in a certain school.

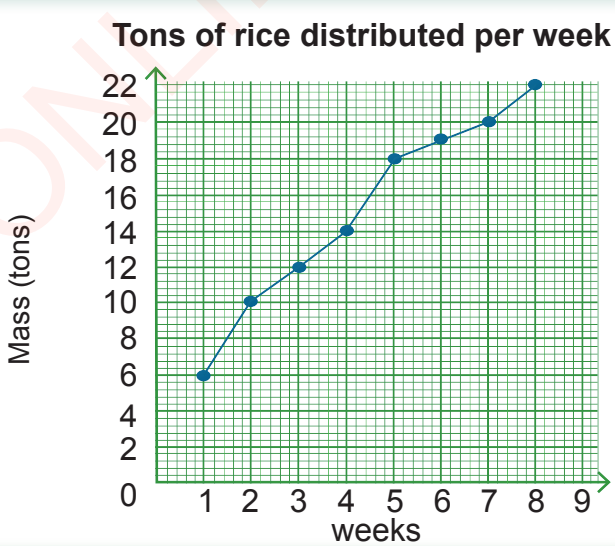


(a) Read the graph, and then fill in the following table:

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of pupils					

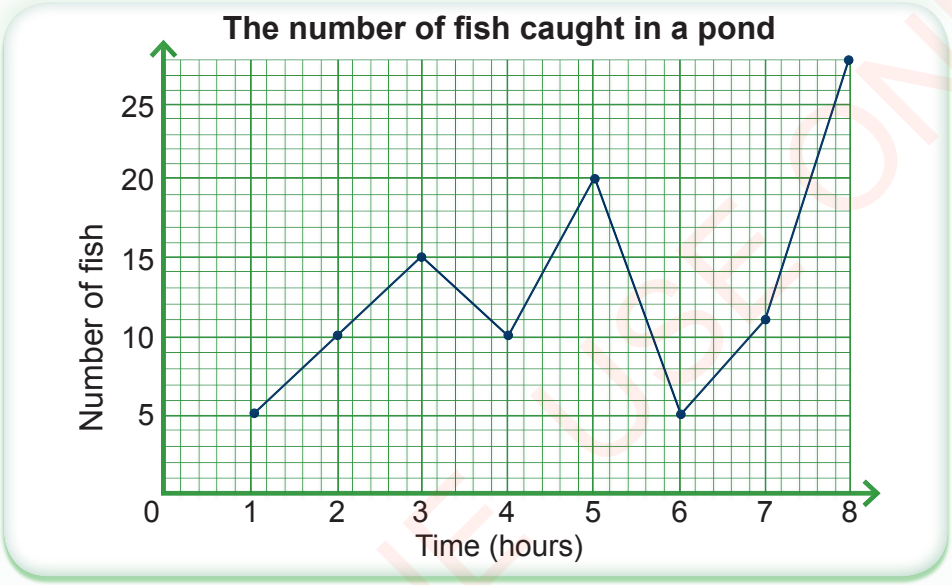
(b) On which day did most of the pupils attend school?
(c) On which day did few pupils attend school?
(d) Mention the days that the same number of pupils attended school.

3. The following graph shows tons of rice distributed per week.



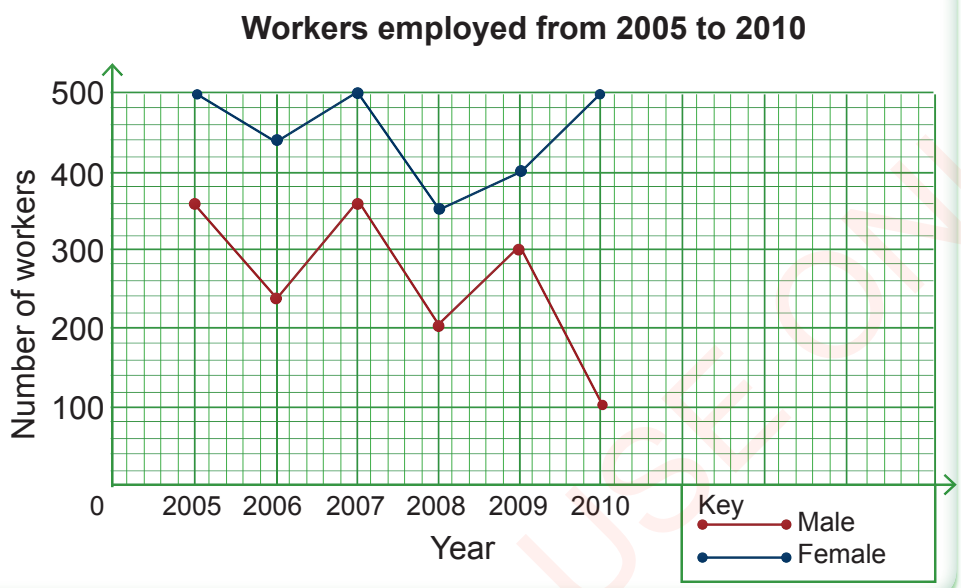
(a) How many tons of rice were distributed in the third week?
(b) How many tons of rice were distributed in the first week?
(c) Find the difference between the number of tons of rice distributed in the second week and the number of tons distributed in the eighth week.
(d) In which week were 14 tons distributed?
(e) How many tons of rice were distributed in the eighth week?

4. The following graph shows the number of fish caught in a pond for 8 hours.



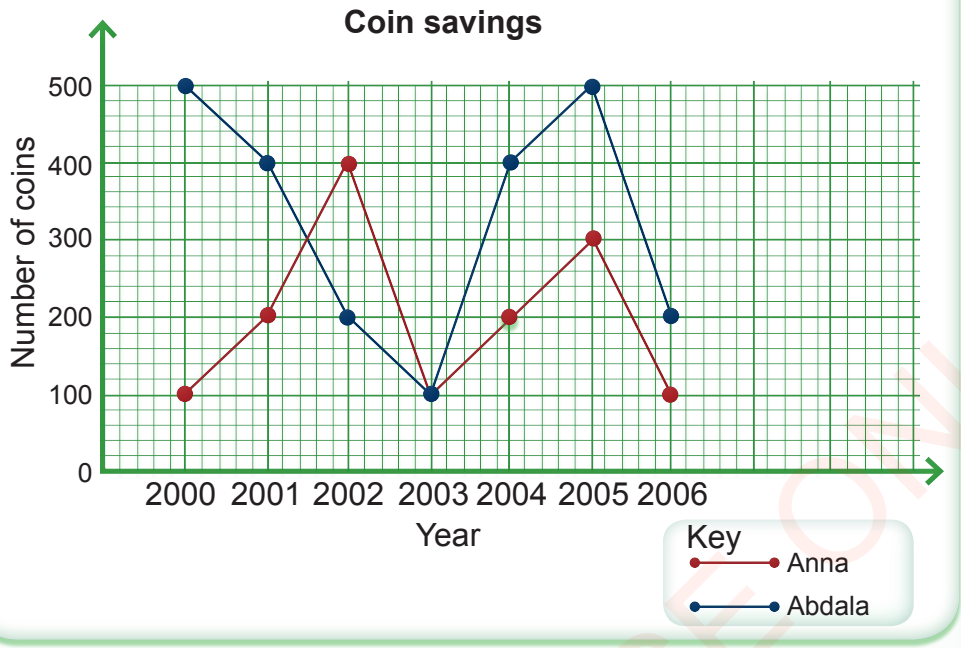
(a) Which hour shows the largest number of fish caught?
(b) Which hour shows the smallest number of fish caught?
(c) Did the number of fish caught from 3 hours to 4 hours increase or decrease? What was the difference in fish caught between those hours?
(d) How many fish were caught in the fifth hour?
(e) What is the difference in number of fish caught between the fifth hour and the eighth hour?
(f) At what time between the first hour and the fourth hour was the least number of fish caught?
(g) Find the difference between the number of fish caught in the fourth hour and the fifth hour.
(h) If x is the total number of fish in 8 hours and y is the number of fish in first hour, find the value of $2x - \left(x + \frac{1}{5}y\right)$.

5. A certain institution has employed male and female workers. The following graph represents their numbers from 2005 to 2010. Study the graph, and then answer the questions that follow:



(a) In which years did the institution hire most female workers?
(b) How many workers were hired in 2005?
(c) In which year did the institution hire a total of 860 workers?
(d) Find the difference between the number of male and female workers hired in 2007.
(e) In which year was the number of male workers higher than the number of female workers?
(f) Find the total number of male and female workers hired in 2010.
(g) Find the total number of male and female workers hired from 2005 to 2007.

6. The following graph shows the number of coin savings for Anna and Abdala from the year 2000 to 2006. Study the graph, and then answer the questions that follow.



(a) Find the number of coins saved by both Anna and Abdala in the year 2000.

(b) Find the difference between the number of coins saved by Anna and those saved by Abdala in the year 2003.

(c) In which year(s) did Anna save more coins than Abdala? By what amount?

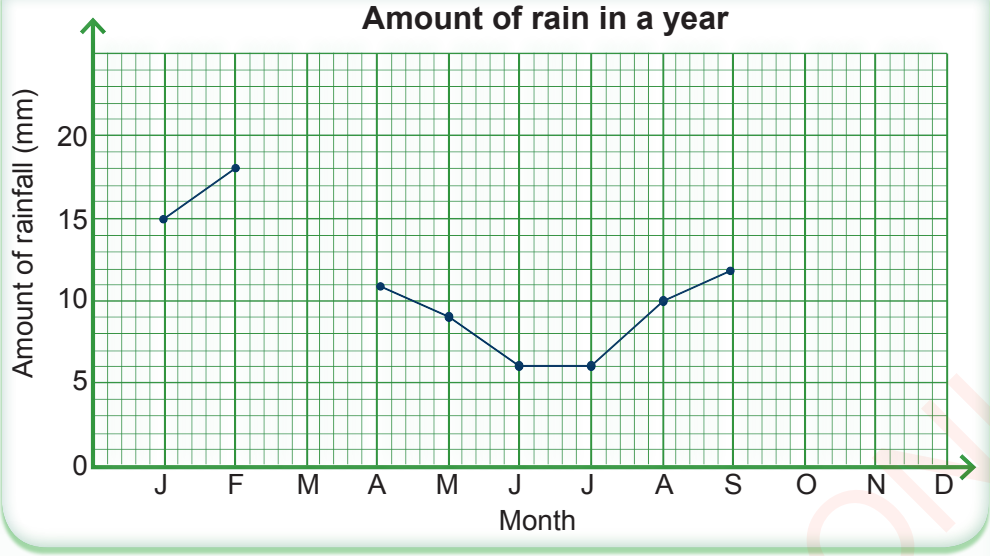
(d) In which year(s) did Abdala save more coins than Anna? By what amount?

(e) What is the number of coins saved by both Anna and Abdala in the year 2003?

(f) Which year did Anna and Abdala save the highest number of coins altogether? How much was saved?

(g) Which year did Anna and Abdala save the least number of coins altogether? How much was saved?

7. A certain region recorded the amount of rainfall in a year. The data are presented in the following graph and table. Study the graph and answer the questions that follow.



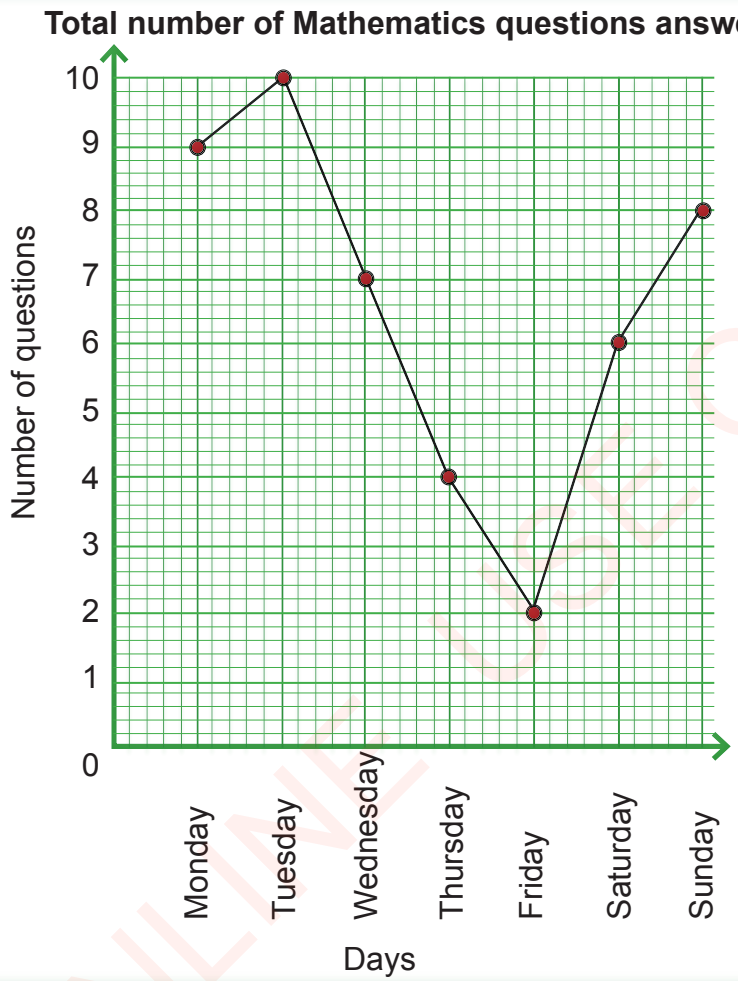
Amount of rainfall in a year

Month	January	February	March	April	May	June	July	August	September	October	November	December
Amount of rainfall (mm)	—	—	14	—	—	—	—	—	—	17	19	16

(a) Fill in the table, and then plot the missing points on the graph and draw a complete graph.
(b) By what amount did the rainfall decrease from February to March?
(c) Which months had the least amount of rainfall?
(d) Find the monthly average amount of rainfall.
(e) Which month had the highest amount of rainfall?
(f) Answer "Yes", "No" or "No idea" in the following questions:

(i)	The rainfall kept decreasing from February to July
(ii)	February is expected to get floods compared to other months
(iii)	The rainfall in September was twice as much as in June
(iv)	The rainfall in February was three times as much as in July
(v)	The rainfall in February was twice as much as that in May

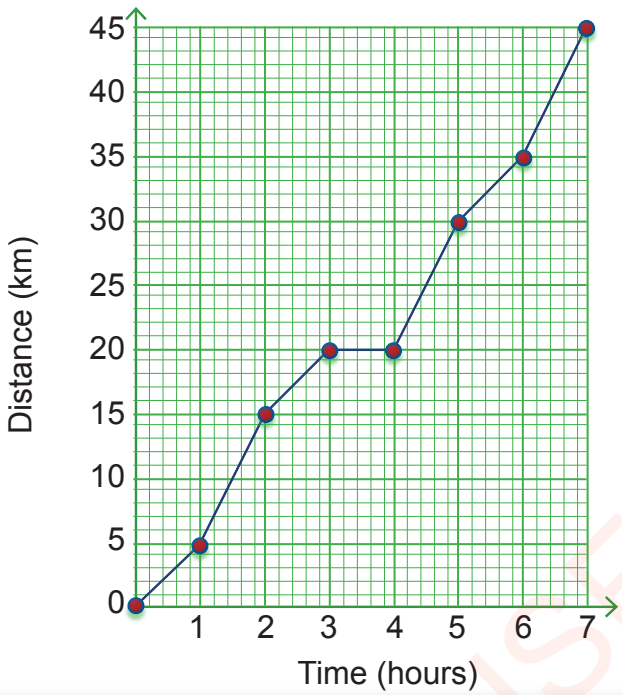
8. The following graph shows the total number of Mathematics questions answered by a pupil in a week. Study the graph, and then answer the questions that follow:



(a) How many questions did the pupil answer on Tuesday?
(b) In which day did the pupil answer 7 questions?
(c) In which day did the pupil answer 9 questions?
(d) In which day did the pupil answer the lowest number of questions?
(e) Find the average number of questions answered in a day.
(f) In which day would you suggest to sit for a test? Give reasons.

9. The following graph shows a bicycle speed for 7 hours. Study the graph, and then answer the questions that follow:

A bicycle's speed for seven hours



(a) Find the distance that the bicycle rider travels in every hour for the whole trip.

(b) Find the speed of the bicycle rider from the third to the fourth hour.

(c) Find the total distance the bicycle rider travelled from the second hour to:

(i) Third hour.

(ii) Fourth hour.

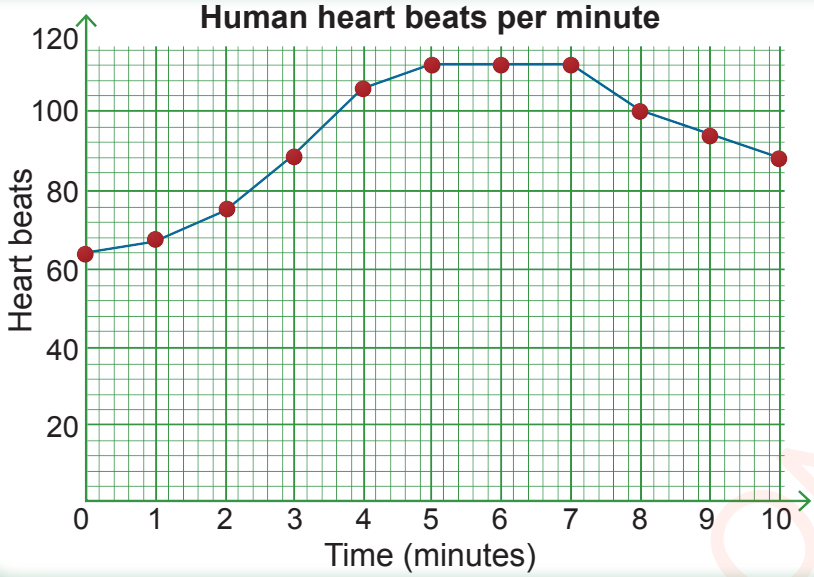
(d) What did you notice from the answers in (c) (i) and (c) (ii)? Explain.

(e) Find the distance covered by the rider for the whole trip.

(f) Briefly describe the trip of the bicycle rider from start to end.

(g) Construct 5 questions from the explanation provided in (f).

10. The following graph shows human heart beats per minute during an exercise. The dots represent changes in heart beat. Answer the following questions:



(a) How long did it take to reach the highest heart beat?

(b) How many heart beats were recorded in the first minute?

(c) How many heart beats were recorded in the $2\frac{1}{2}$ minute?

(d) At what time do you think the exercise ended?

Drawing a line graph

Example

The following table shows the number of litres of milk produced in a dairy factory and its cost in Tanzanian shillings.

Milk (litre)	1	2	3	4	5	6	7	8
Cost (shs)	2 000	4 000	6 000	8 000	10 000	12 000	14 000	16 000

Draw a line graph to represent this information.

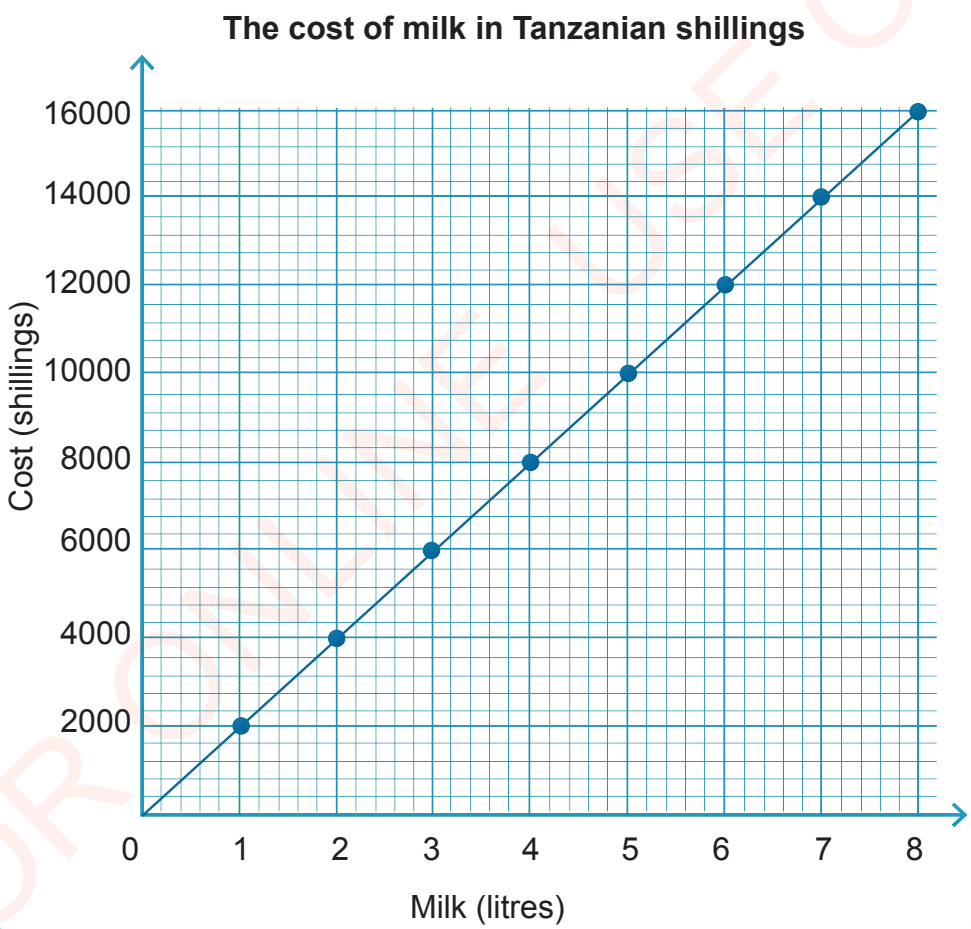
The following are important steps when drawing a line graph:

(a) Prepare a graph by drawing the vertical line (y-axis) and the horizontal line (x-axis) to present the intended information. Write the heading of the graph that reflects the information presented.

(b) Identify and arrange all information that will be on the y-axis and on the x-axis. Choose the scale to be used. For instance, on the y-axis 1 cm represents 2 000 shillings and on the x-axis 1 cm represents 1 litre.

(c) Use the information presented in the table to mark the dots in the graph. For example, the price of 1 litre of milk is 2 000 shillings.

(d) Connect the dots by a straight line. The resulting line is called a line graph which represents the information from the table.



Activity: A project on various statistics

Materials: A pen and paper.

Procedure:

1. At the school or home environment, gather five kinds of information on different types of trees and their respective numbers.
2. Write the information in the following table.

Types of trees					
Total					

3. Draw a line graph by using the information presented in the table in step 2.
4. Analyse and interpret the graph drawn in step 3.

Exercise 3

Answer the following questions:

1. The following table shows the amount of rainfall from March to June.

Month	March	April	May	June
Amount of rainfall (mm)	30	35	20	15

(a) Use the data in the given table to draw a line graph.
(b) Which month had the least amount of rainfall?
(c) Which month had the highest amount of rainfall?
(d) Find the month with 15 mm of rainfall.
(e) Calculate the monthly average rainfall.

2. The following table shows the amount of paddy harvested by villagers in five years. Draw a line graph to represent this information.

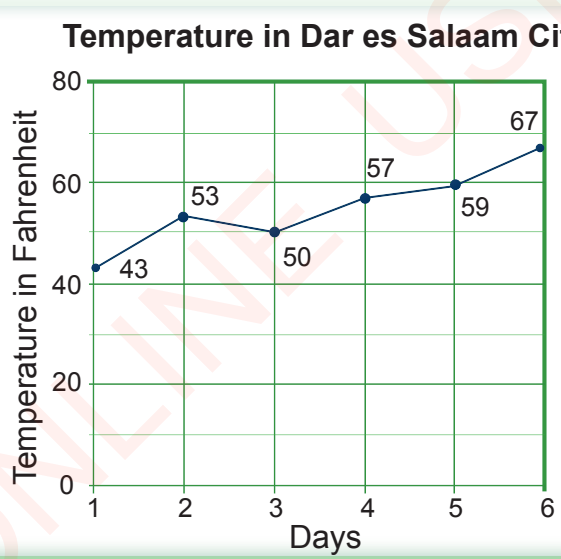
Year	2014	2015	2016	2017	2018
Mass (tons)	160	120	180	240	300

3. The following table shows the amount of time used to empty a water tank:

Time (minutes)	0	1	2	3	4	5	6	7	8
Amount of water (litres)	160	140	120	100	80	60	40	20	0

(a) Draw a line graph to show the amount of water in the tank against time.
(b) Suppose that after empting the tank, the pupil was allowed to go for self study. After how long did the pupil go for self study?

4. Study the line graph, and then answer the questions that follow.



(a) Draw a table to represent the information in the line graph.
(b) What was the temperature on the third day?
(c) Which day had the lowest temperature?
(d) Pupils were required to wear sweaters on two different days. Mention the days and state the reason for wearing sweaters.
(e) Pupils were required to open all windows for two days consecutively. Mention the days and state the reason.

5. The following table shows how Minja rode a motorbike in six days. Study the following table, and then draw a line graph.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Distance (km)	64	88	112	128	104	80

6. Maendeleo and Mafanikio primary schools collected plastic bottles during the environmental conservation campaign as shown in the following table. Study the table, and then draw a line graph.

Year	Number of plastic bottles	
	Maendeleo Primary School	Mafanikio Primary School
2010	800	600
2011	700	500
2012	800	700
2013	900	600
2014	1000	700
2015	900	800

7. The following table represents Mathematics text books sales in a stationery. Study the table, draw a line graph, and then answer the questions that follow.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of customers	45	30	5	20	25	50

(a) Which day had the highest number of customers?
(b) Which day had the least number of customers?
(c) Find the difference between the days with the highest and the least number of customers.
(d) Find the average number of customers per day.

(e) If the stationery owner decides to buy gifts for the customers, how many gifts should the owner buy so that each customer gets one?

(f) If the stationery owner decides to reduce the working days from six to five, which day should the owner close the shop? Why?

8. The following table shows Naomi's height from the age of 2 years to 16 years. Study the table, and then answer the questions that follow.

Age (years)	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Height (cm)	86	94	102	107	112	117	127	137	145	155	160	163	165	168	170

(a) Draw a line graph to represent Naomi's height every year.

(b) At which age did Naomi reach the height of 1.02 metres?

(c) Naomi joined standard one at Uhuru Primary School when she had the height of 1 metre and 12 centimetres. At what age did Naomi join the school?

(d) Naomi shifted to Umoja Primary School in Standard Five with the height of 145 cm. How old was she?

(e) Answer "Yes", "No" or "No idea":

(i)	Naomi's height increased slowly between the age of 5 and 6 years
(ii)	Naomi's height increased faster between the age of 2 and 3 years
(iii)	Naomi completed primary school at the age of 12 years
(iv)	Naomi's height when she completed primary school can be obtained

9. The following table shows the distance ran by an athlete and the time used. Study the table, draw a line graph and answer the questions that follow.

Time (hours)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
Distance (km)	0	9.6	16	24	30.4	35.2	38.4

(a) How long will it take for the athlete to run 19.2 kilometres?

(b) Briefly explain the trend of the speed as the athlete continues to run.

10. The following table shows Standard Seven pupil's results for six subjects. Study the table and then answer the questions that follow.

Subject	Mathematics	English	Kiswahili	Science and Technology	Civic and morals	Vocational skills
Marks (%)	86	78	88	80	72	68

(a) Draw a line graph to represent the information given in the table.

(b) Which subject did the pupil perform the best?

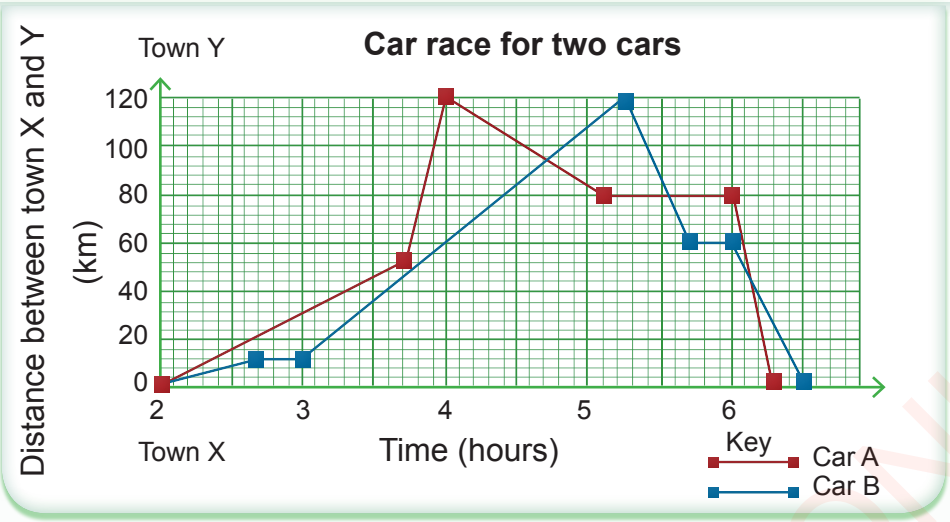
(c) Which subject did the pupil score the lowest marks?

(d) Find the average mark the pupil scored in all the subjects.

Revision exercise

Answer the following questions:

1. The following graph shows a race between car A and B from town X to town Y, then back to town X:



(a) How far is town Y from town X?

(b) At 3:30 pm, which car was leading the race?

(c) At 5:30 pm, which car was leading the race?

(d) At 4:00 pm, what was the distance between the two cars?

(e) Which car won the race?

(f) What was the time difference of finishing the race between car A and B?

(g) What was the time difference of the two cars in reaching town Y?

(h) Which car stopped twice during the race? At what time?

(i) Did one car overtake the other? If yes, at what time?

2. The following table shows the number of tourists in a national park. Study the table, and then answer the questions that follow.

Day	Friday	Saturday	Sunday	Monday
Number of tourists	697	2 115	2 346	465

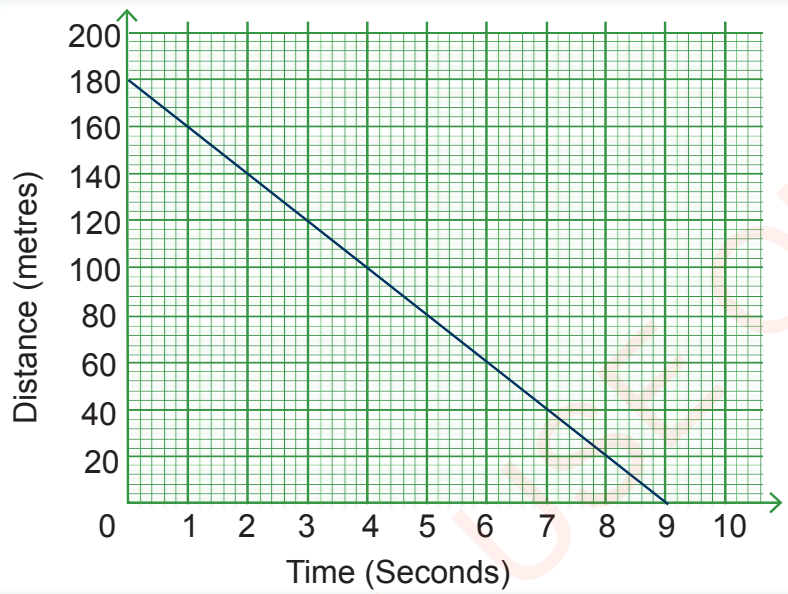
(a) Use the given data in the table to draw a line graph.

(b) Which day had the highest number of tourists? Give reasons.

(c) Which day had the least number of tourists? Give reasons.

(d) If every tourist paid 6 000 shillings, find the total income from the tourists for four days.

3. Study the following graph, and then answer the questions that follow.



(a) Suggest the title for the graph and give a short explanation from the graph.

(b) Tabulate the data given in the graph.

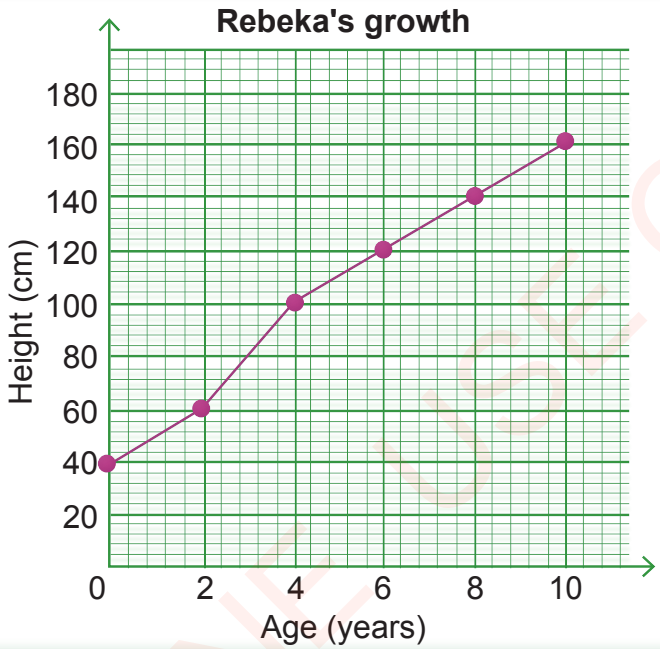
4. (a) The following table shows the time in minutes and sunlight duration for eight days. Approximate the data to the nearest hundreds, and then use the approximated data to draw a line graph.

Day	First	Second	Third	Fourth	Fifth	Sixth	Seventh	Eighth
Time (minutes)	360	210	300	600	480	350	720	160

(b) The following table shows the amount of money saved in a month. Use the table to draw a line graph by approximating the data to the nearest hundreds.

Month	January	February	March	April	May	June
Shillings	5 125 000	19 050 000	6 120 000	4 025 000	2 600 000	6 975 000

5. The following line graph shows Rebeka's growth (in height) from her first to tenth year. Use the graph to answer the questions that follow.



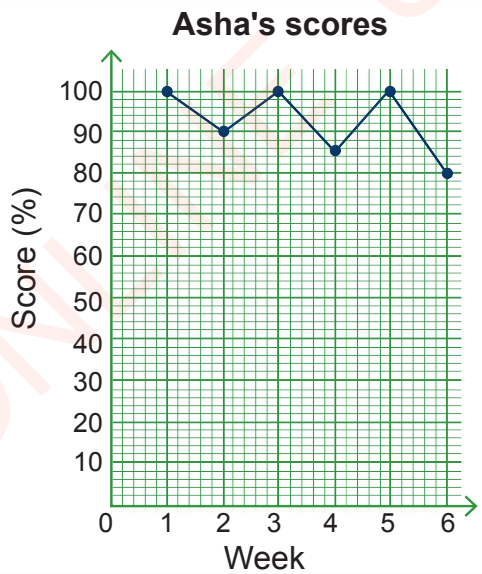
(a) What was Rebeka's height when she was born?
(b) What was Rebeka's height at the age of three years?
(c) How old was Rebeka when she had the height of 120 cm?
(d) How old was Rebeka when she had the height of 1 metre?
(e) At what age did Rebeka's height increase rapidly during her growth period?
(f) How much did Rebeka's height increase between the age of 5 and 6 years?
(g) What was Rebeka's height at the age of 10 years if her growth rate remained unchanged from the age of 4 years?
(h) Can you approximate Rebeka's height at the age of 60? Give a reason.

6. In order to perform well in Mathematics, Joshua solves Mathematics questions everyday as shown in the table below. Use the table to answer the questions that follow.

Days	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Number of questions	36	26	20	24	12	48	44

(a) Draw a line graph to represent this information.
(b) How many questions does Joshua solve on Saturdays and Sundays?
(c) How many questions does Joshua solve from Monday to Tuesday?
(d) Find the number of questions he solves in one week.
(e) Find the average number of questions Joshua solves per day.

7. The following graph represents the marks of Mathematics tests obtained by Asha in 6 weeks. Use the graph to answer the questions that follow.

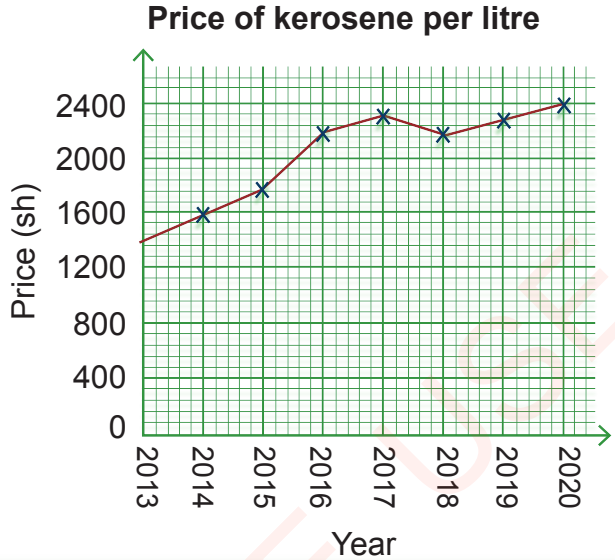


(a) What did Asha score in the first week?
(b) What was Asha's total score in the last three weeks?

(c) If the pass mark is 90%, did Asha pass or fail? Give reasons.

(d) In three different weeks, Asha scored low marks, what are those weeks? How much did she score?

8. The following graph shows the annual price of kerosene per litre in Mbeya City. Study the line graph, and then answer the questions that follow.



(a) Fill in the following table by using the given line graph.

Year	2013	2014	2015	2016	2017	2018	2019	2020
Price (sh)								

(b) What was the price for one litre of kerosene in 2017?

(c) Did the price of kerosene rise or fall in 2019 compared to that of 2016? By how much?

(d) In which years was the price of kerosene the same?

(e) If the rise in price of kerosene from 2013 to 2015 was the same as that of the past two years, find the price of kerosene in the year 2011 and 2012.

(f) If the rise in price of kerosene from 2018 to 2020 will be the same in the next three years, find the price of kerosene in the year 2021, 2022 and 2023.

9. The following table represents the number of absent pupils in Standard One to Standard Five in one month. Study the table, and then answer the questions that follow.

Standard	I	II	III	IV	V
Number of absentees	11	2	14	2	5
Percentage					

(a) Find the percentage of absentees in each class.
(b) Draw a line graph showing the number of absentees in all classes.
(c) You have been invited to advise the pupils from two classes among the five concerning regular school attendance at school. Which classes will you choose first? Give reasons.

10. In your own words, explain by using examples how you will use a line graph to solve different issues in daily life.

Summary

1. A line graph is an important tool that represents the relationship of variables.
2. Line graphs can be used to represent relationship of various types of information.
3. A line graph is read and interpreted by using the horizontal line (x-axis) and the vertical line (y-axis).

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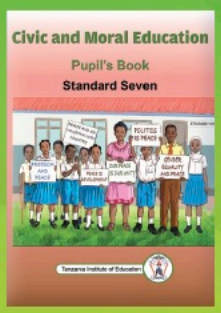
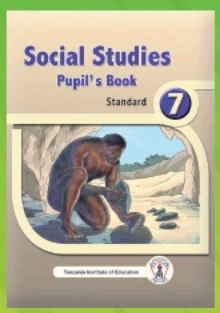
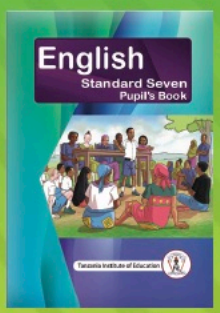
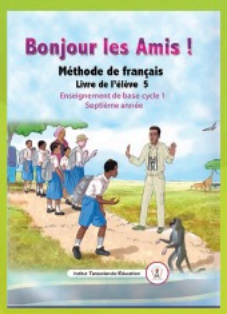
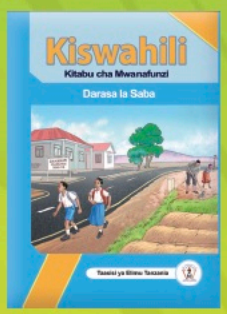
Other Standard Seven Pupil's Books

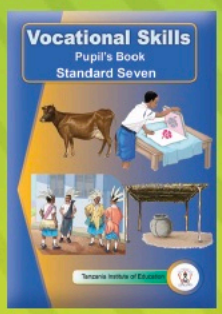


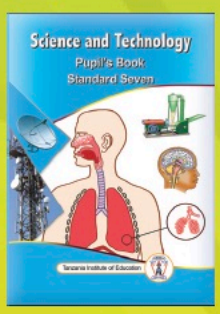












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